UNIVERSITY OF MARYLAND

Using classical mechanics to engineer quantum shortcuts to adiabaticity

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Setup & notation

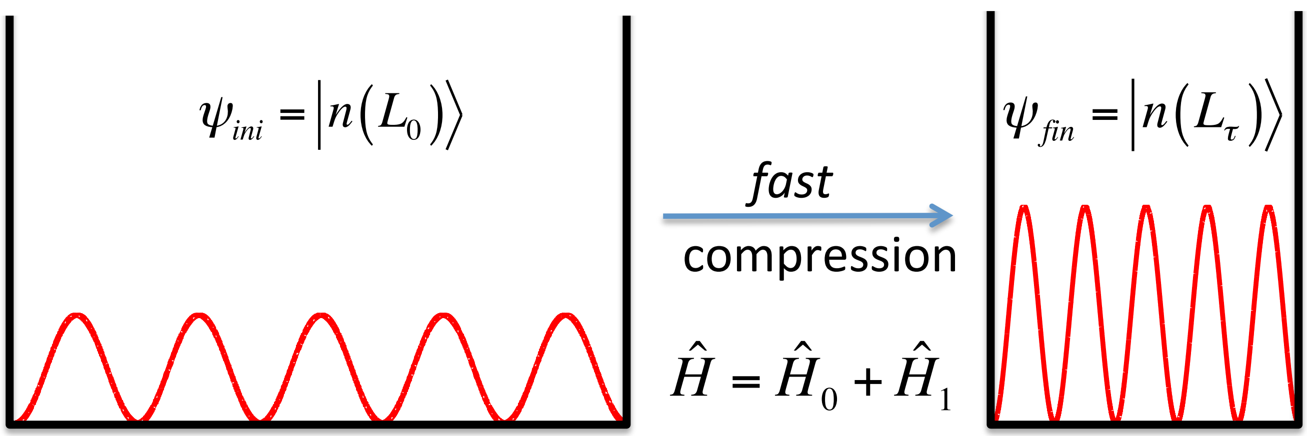
Hamiltonian $\hat{H}_0(\lambda)$ $\quad \hat{H}_0|n\rangle = E_n|n\rangle$

protocol λ_t $\quad 0 \le t \le \tau$

goal: $\psi_{ini} = |n(\lambda_0)\rangle \rightarrow \psi_{fin} = |n(\lambda_\tau)\rangle$

$$\hat{H}(t) = \hat{H}_0(\lambda_t) + \hat{H}_1(t)$$

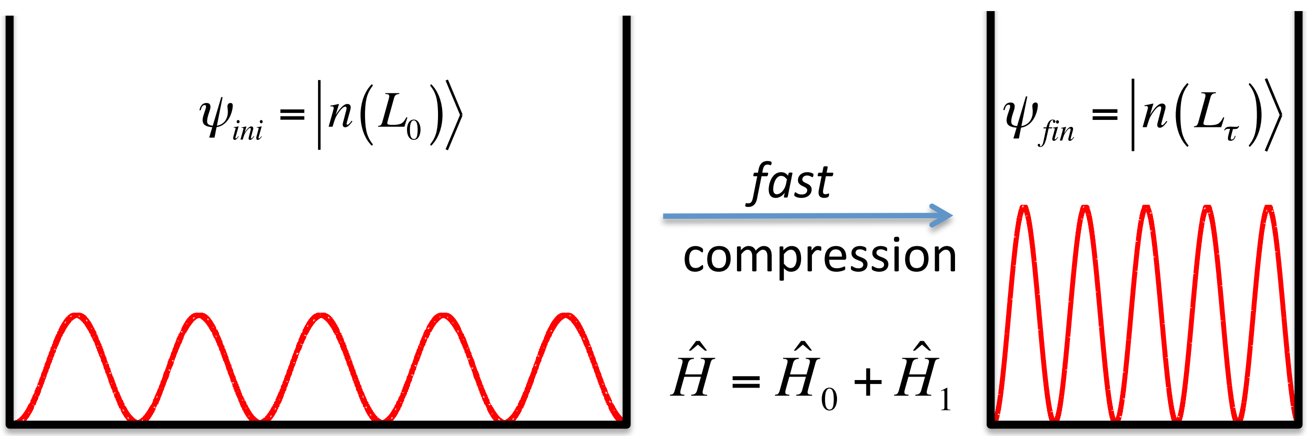
← counter-diabatic term



Setup & notation

- <u>strong version</u>: $\psi(t) = |n(\lambda_t)\rangle$ at all times *non-local*
- <u>weak version</u>: $\psi_{ini} = |n(\lambda_0)\rangle$ and $\psi_{fin} = |n(\lambda_\tau)\rangle$ *local*

solve for the counter-diabatic term, $H_1(t)$

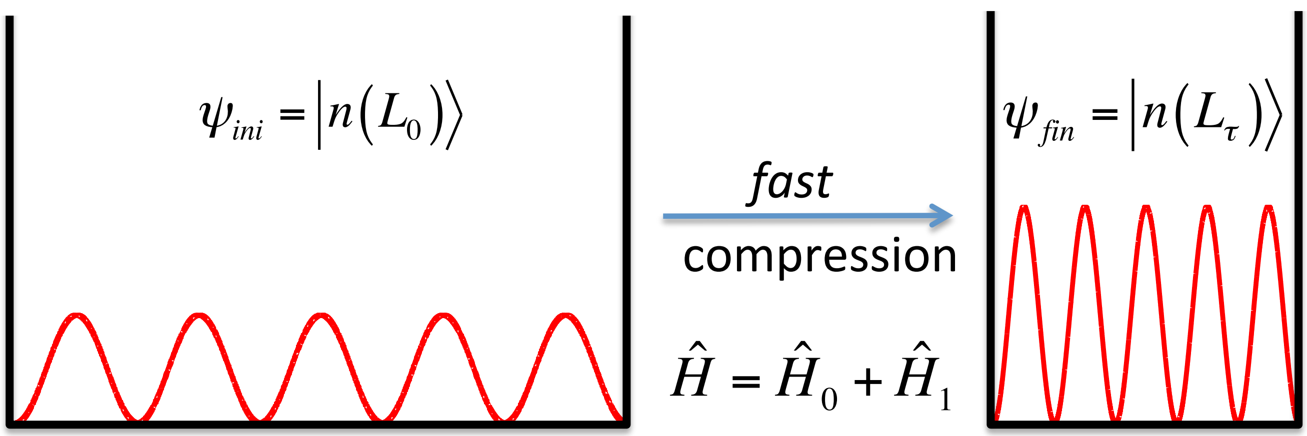


Strong (non-local) shortcuts - transitionless quantum driving

M Demirplak & SA Rice, *J Phys Chem* **107**, 9937 (2003)
MV Berry, *J Phys A* **42**, 365303 (2009)

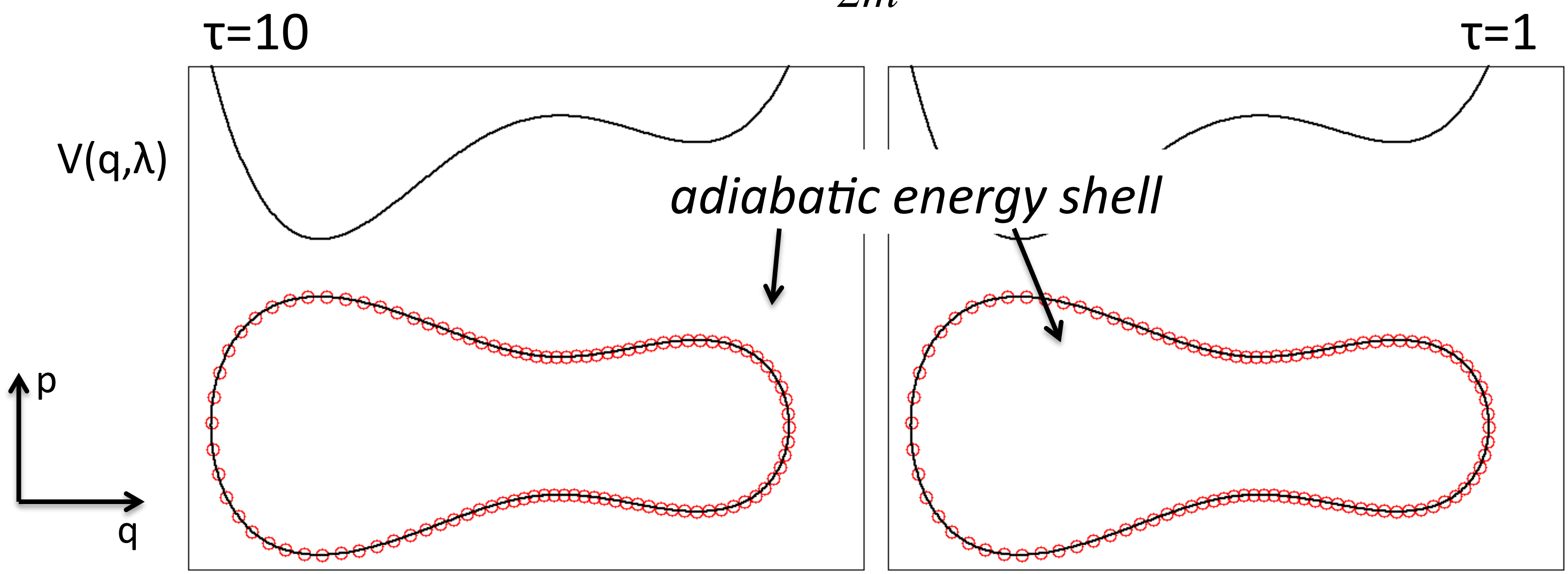
$$|\nabla m\rangle \equiv \frac{\partial}{\partial \lambda}|m(\lambda)\rangle$$

$$\hat{H}_1(t) = i\hbar\dot{\lambda}\sum_m \left(|\nabla m\rangle\langle m| - \langle m|\nabla m\rangle|m\rangle\langle m|\right) \overset{?}{=} H_1(\hat{q},\hat{p},t)$$



Classical adiabatic theorem

1 degree of freedom $$H_0 = \frac{p^2}{2m} + V(q,\lambda)$$



(1) Strong shortcuts: construct a counter-diabatic term $H_1(q,p,t)$ such that trajectories cling to the adiabatic energy shell when evolving under $H = H_0 + H_1$.

(2) Quantize.

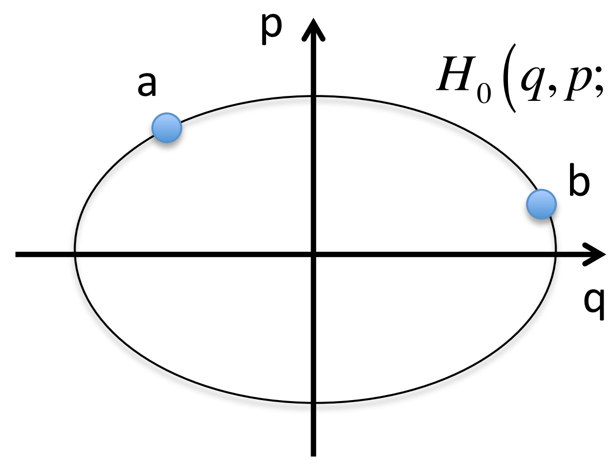
Solving for the classical counter-diabatic term $H_1(q,p,t_0)$:

C.J., *Phys Rev A* **88**, 040101(R) (2013)

1. Necessary & sufficient condition for H_1:

$$\left\{H_1, H_0\right\} = \dot{\lambda}\left(\nabla H_0 - \left\langle \nabla H_0 \right\rangle\right) \equiv A\left(q,p,t_0\right)$$

2. Solution of this equation:

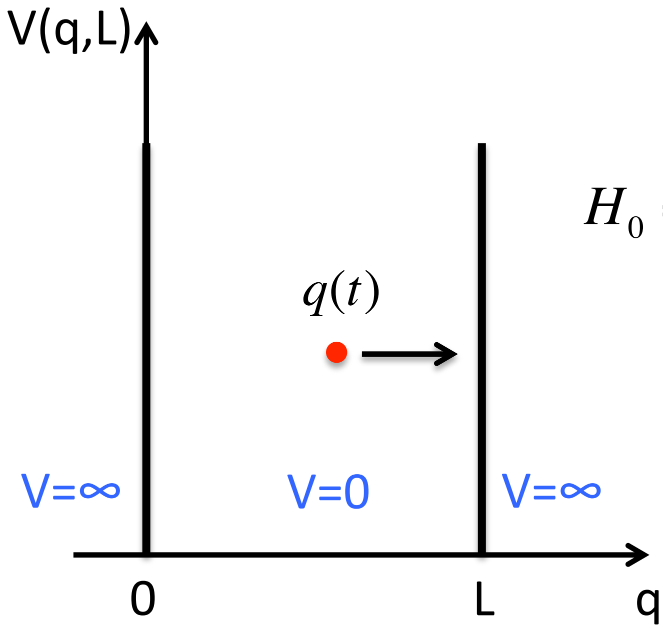


$$H_0\left(q,p;\lambda\left(t_0\right)\right) = E$$

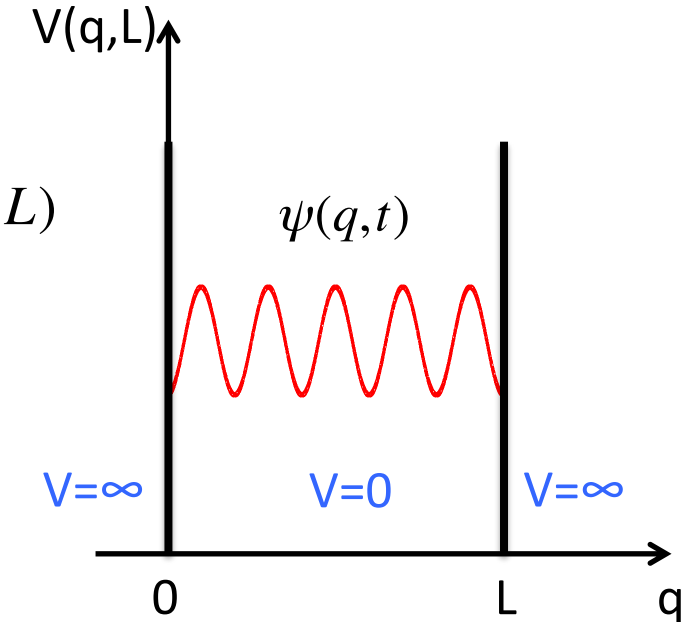
$$H_1\left(b,t_0\right) - H_1\left(a,t_0\right) = \int_a^b dt\ A\left(q(t),p(t),t_0\right)$$

3. Convention: $\left\langle H_1 \right\rangle = 0$

Example #1: particle in box (*piston*), λ=L



$$H_0 = \frac{p^2}{2m} + V(q;L)$$



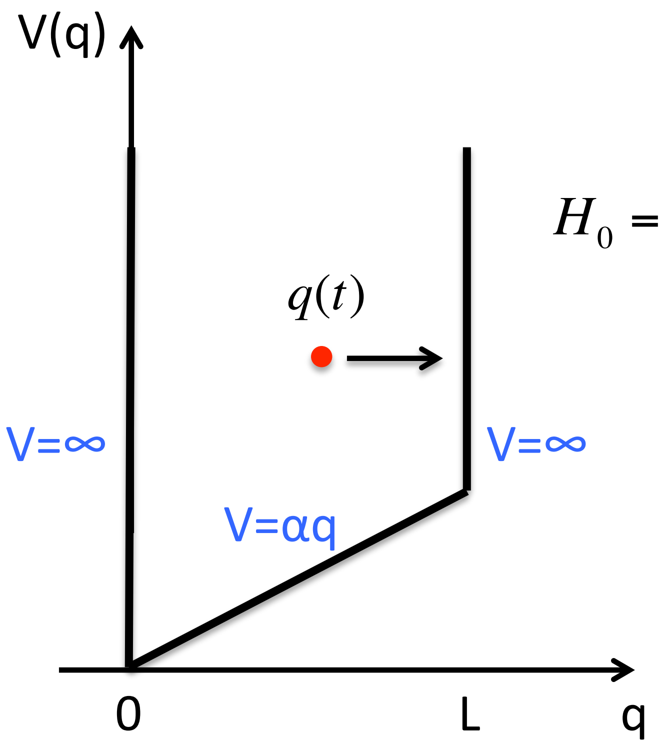
$$H_1 = \frac{\dot{L}}{L} qp$$

$$\hat{H}_1 = \frac{\dot{L}}{2L}\left(\hat{q}\hat{p} + \hat{p}\hat{q}\right)$$

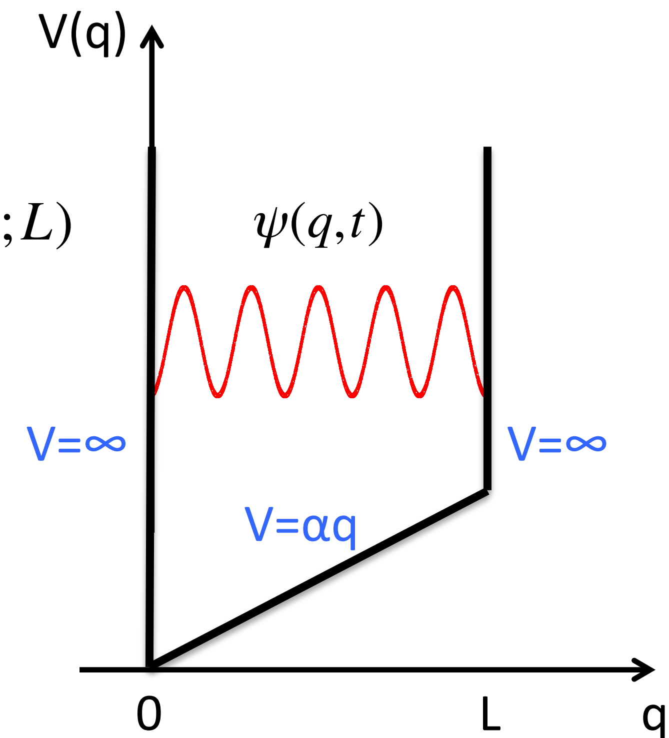
The wavefunction $\psi(q,t) = \sqrt{\frac{2}{L_t}} \sin\left(\frac{n\pi q}{L_t}\right) \exp\left(-\frac{i}{\hbar}\int_0^t dt' \, E_n(t')\right)$

is an *exact* solution of the time-dependent Schrödinger equation, under the Hamiltonian $\hat{H}(t) = \hat{H}_0 + \hat{H}_1$, for arbitrary L(t).

Example #2: *tilted piston*, λ=L



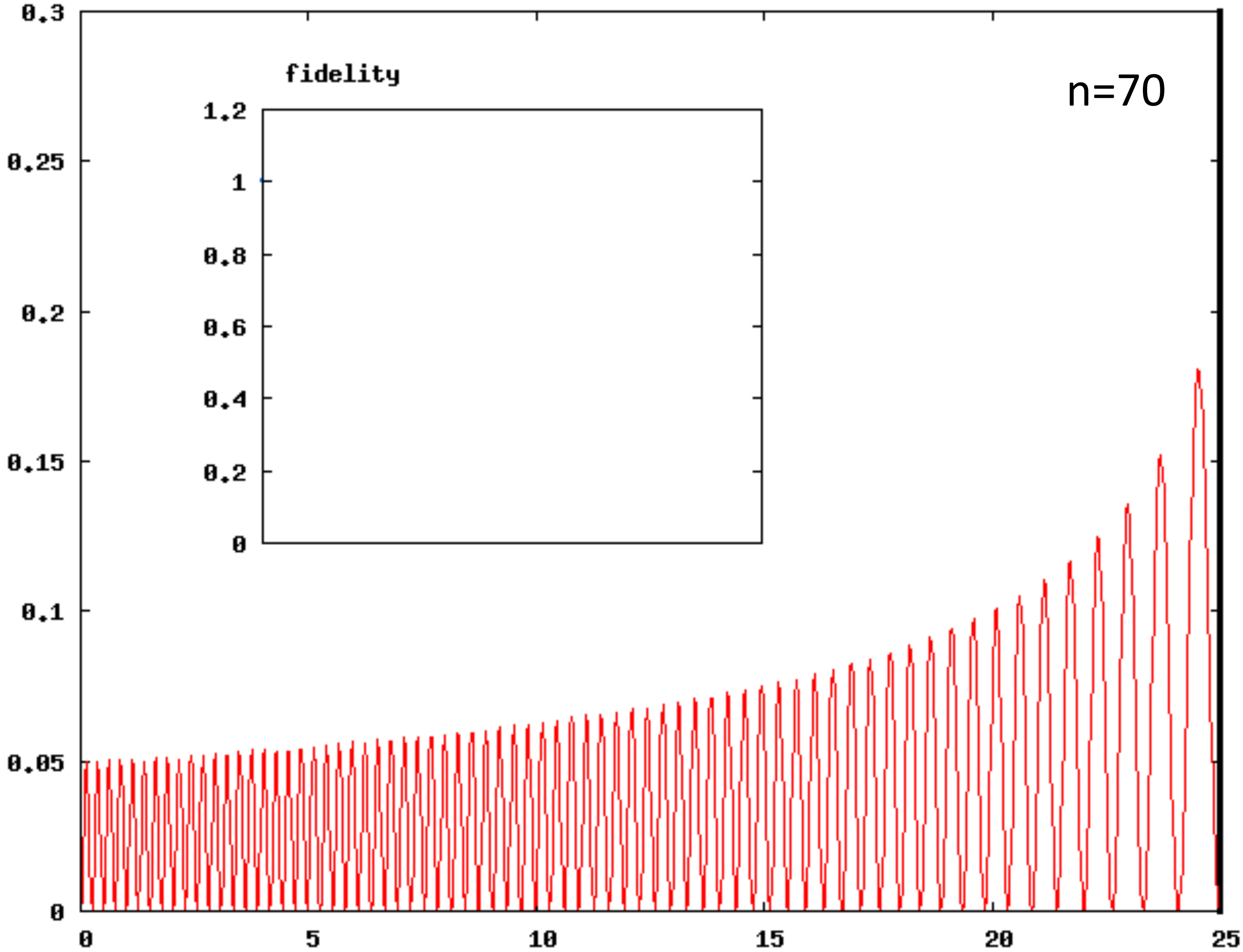
$$H_0 = \frac{p^2}{2m} + V(q;L)$$



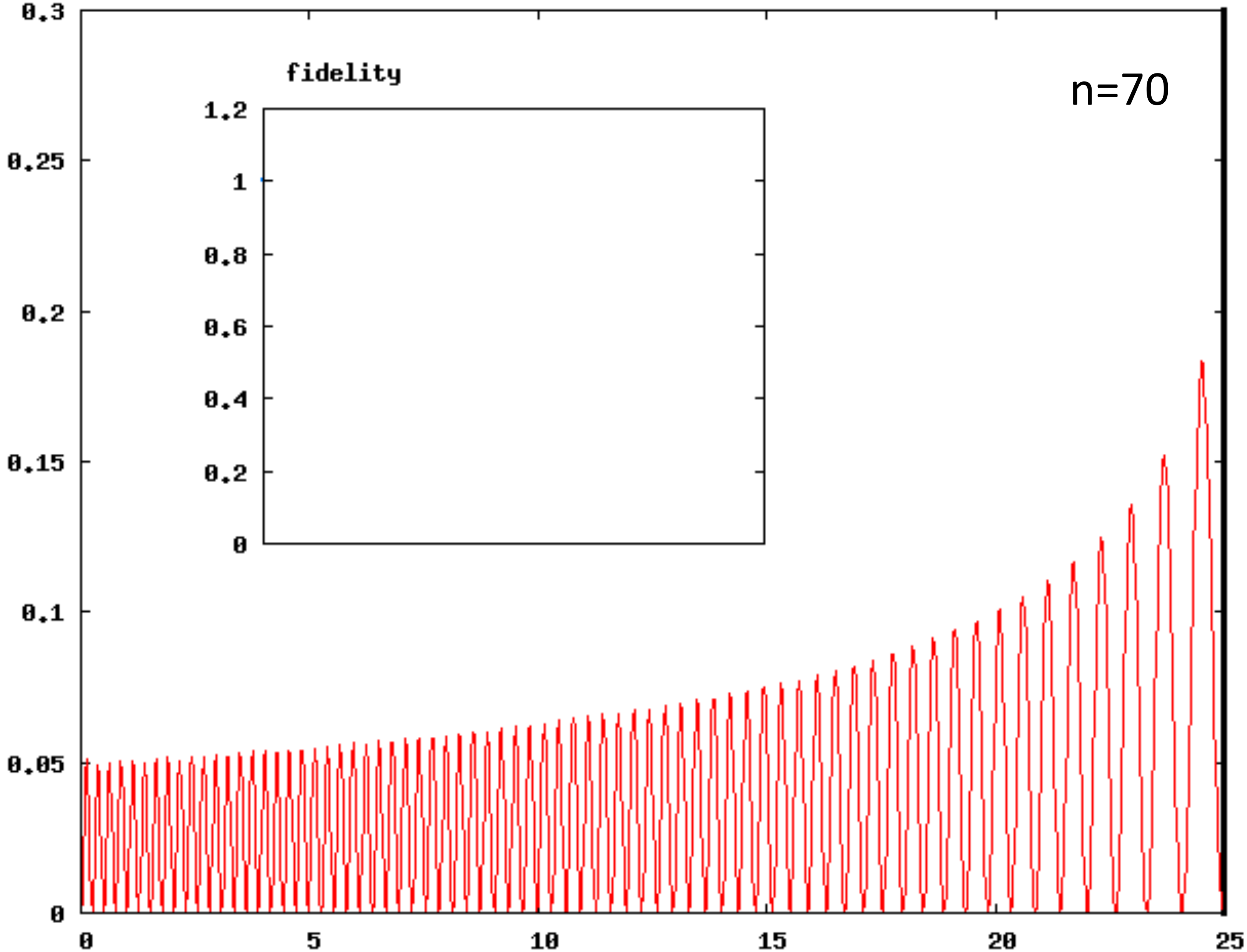
$$H_1 = -\frac{p\dot{L}}{\alpha L}\left[H_0 - \alpha L + \sqrt{H_0\left(H_0 - \alpha L\right)}\right]$$

$$+sign\left(p\right)\frac{\sqrt{2m}\dot{L}}{\alpha L}\left[H_0\sqrt{H_0 - \alpha L} + \sqrt{H_0}\left(H_0 - \alpha L\right)\right]$$

Tilted Piston – $|\psi(q,t)|^2$ *without* counter-diabatic term

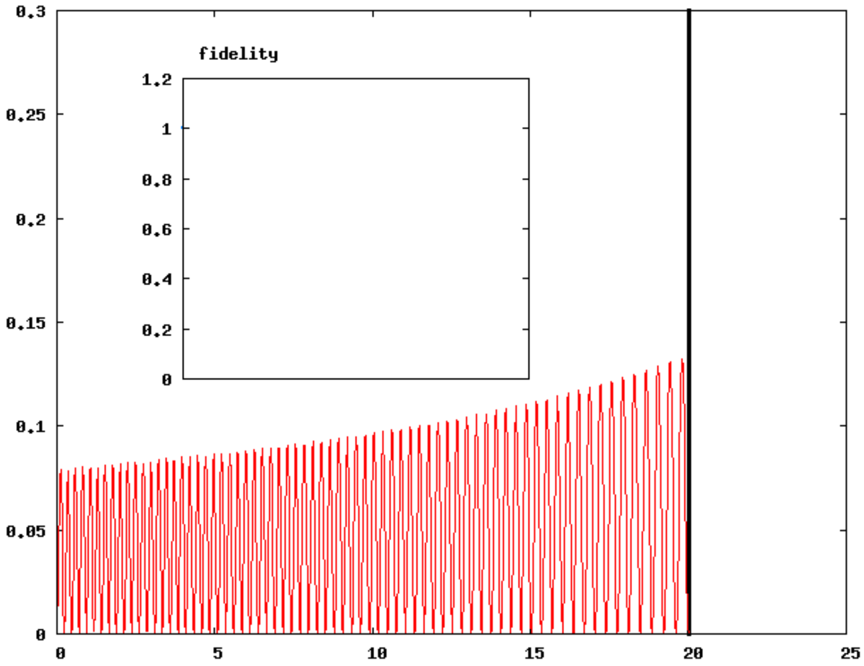


Tilted Piston – $|\psi(q,t)|^2$ *with* counter-diabatic term

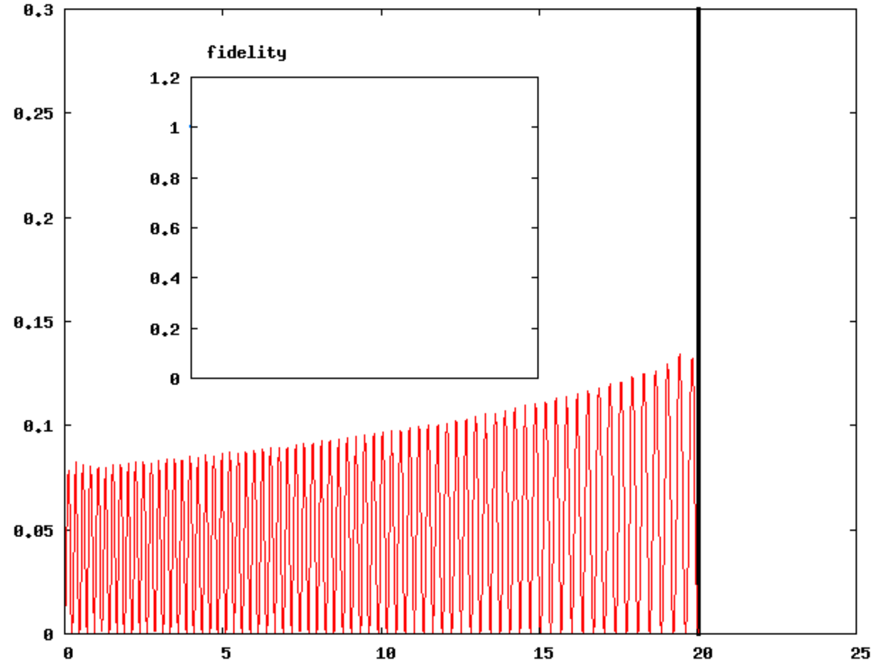


Tilted Piston

$|\psi(q,t)|^2$ *without* CD term



$|\psi(q,t)|^2$ *with* CD term



$$H_1 = -\frac{p\dot{L}}{\alpha L}\left[H_0 - \alpha L + \sqrt{H_0\left(H_0 - \alpha L\right)}\right]$$

$$+ sign(p)\frac{\sqrt{2m}\dot{L}}{\alpha L}\left[H_0\sqrt{H_0 - \alpha L} + \sqrt{H_0}\left(H_0 - \alpha L\right)\right]$$

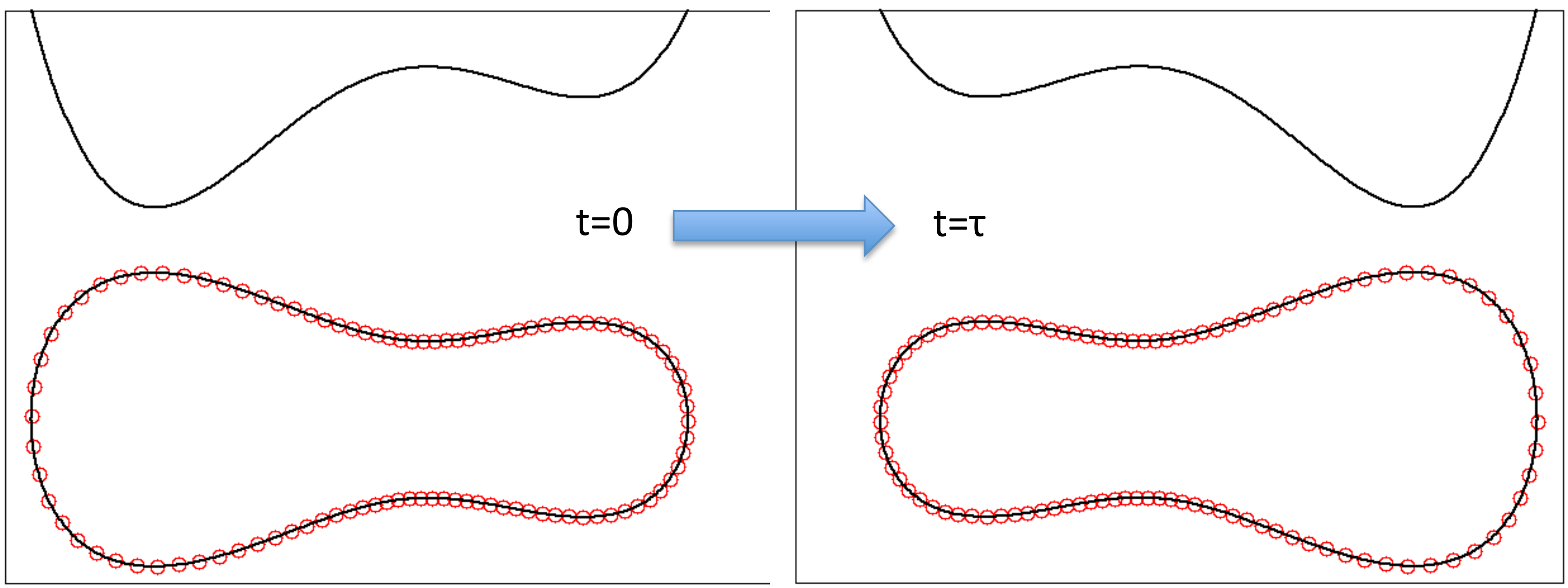
Weak (local) shortcuts to adiabaticity

Fast-forward

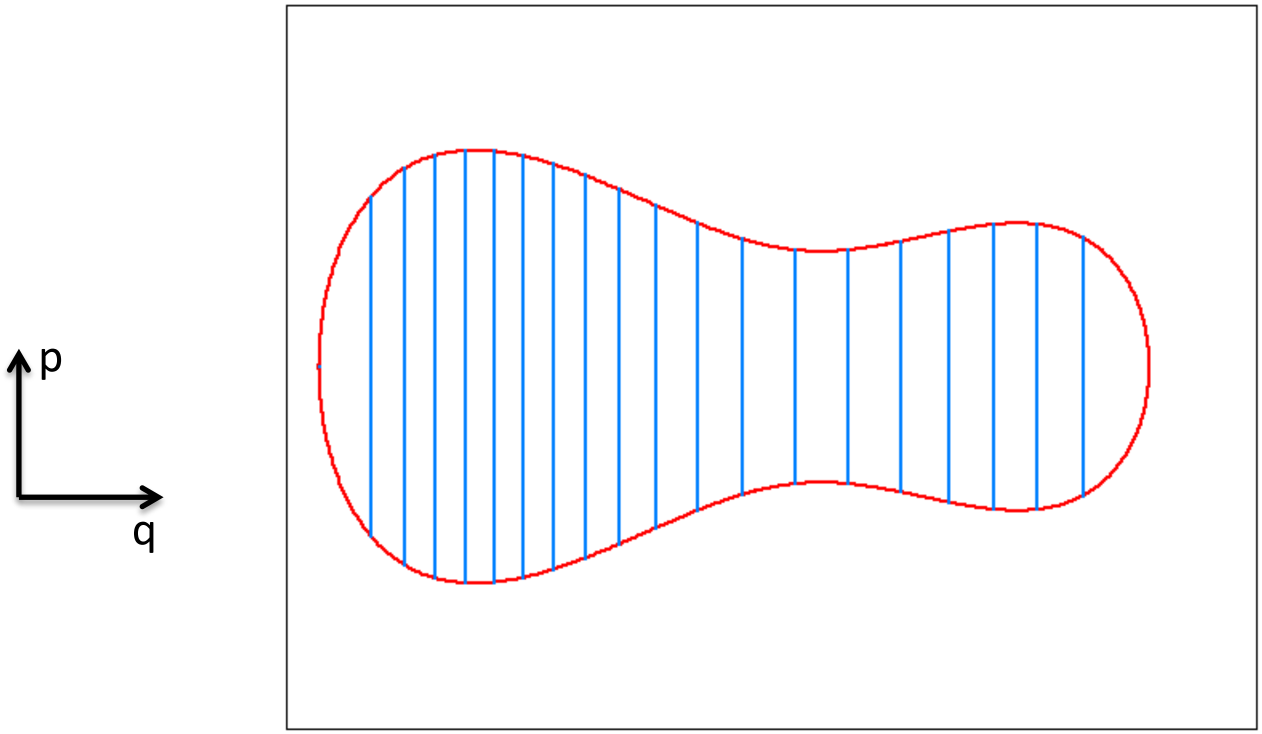
S. Masuda & K. Nakamura, *Phys Rev A* **78**, 062108 (2008)
S. Masuda & K. Nakamura, *Proc Roy Soc A* **466**, 1135 (2010)

Previous goal: find $H_1(q,p,t)$ such that trajectories cling to the adiabatic energy shell at all times. This can be solved, but H_1 is generally *non-local*.

New goal: find $H_1(q,p,t)$ such that trajectories *begin and end* on the adiabatic energy shell. This can be solved with a *local* term: $H_1(q,p,t) = V_1(q,t)$.



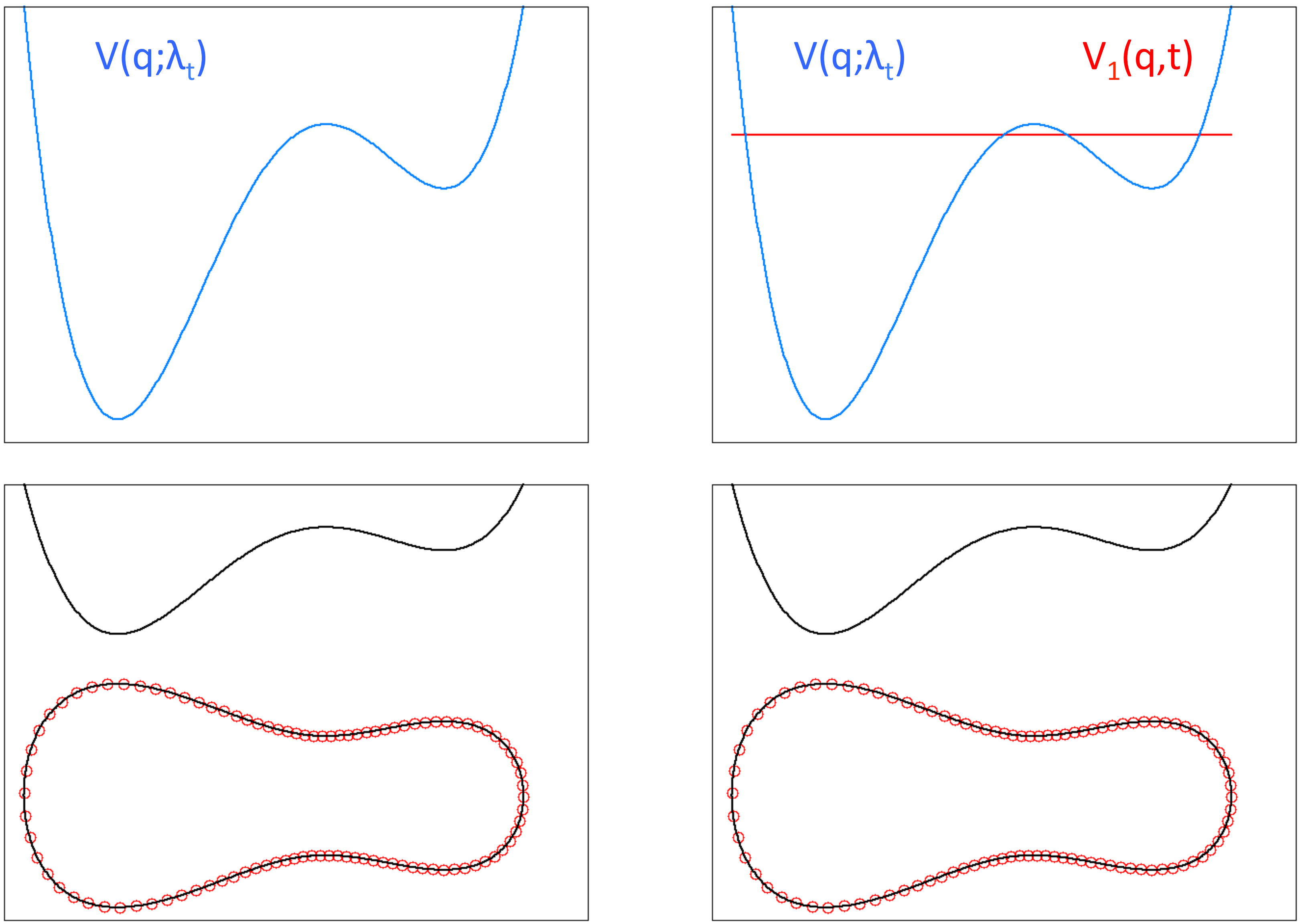
Construction of $V_1(q,t)$ (given H_0 , λ_t)



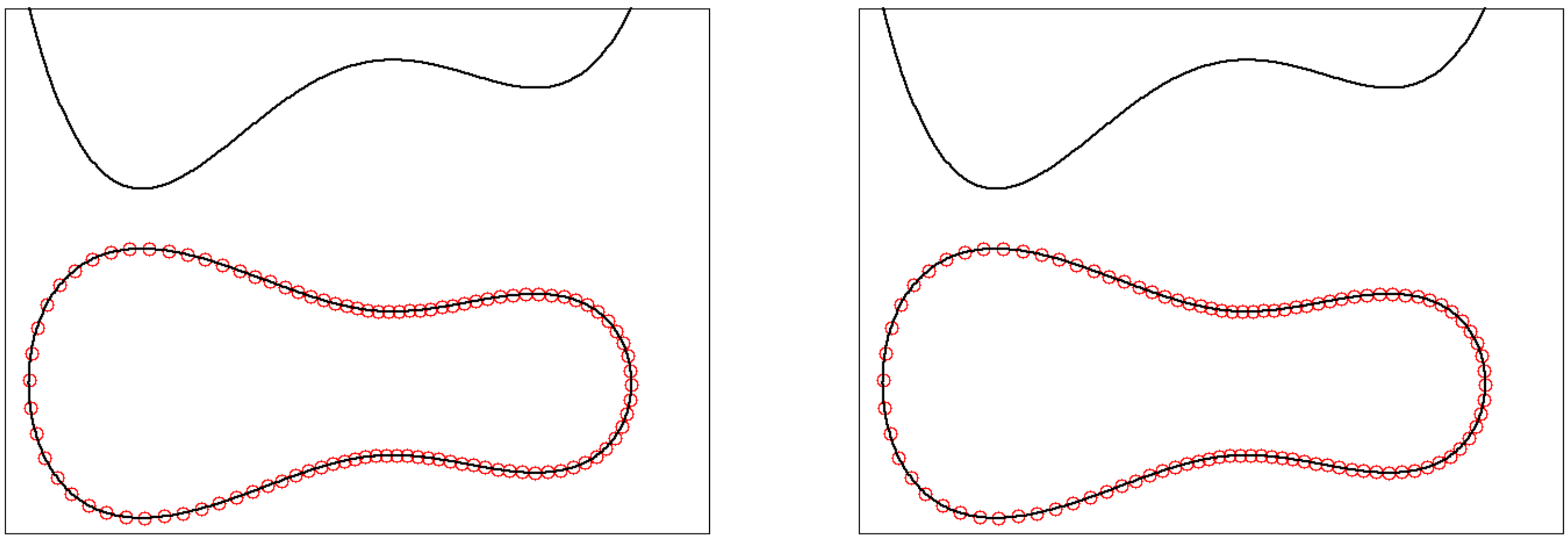
- vertical lines at $q_1(\lambda)$, $q_2(\lambda)$ … $q_{N-1}(\lambda)$ divide the adiabatic energy shell into N strips of equal phase space volume ($N \to \infty$)
- $a(q,t)$ = acceleration field describing the motion of these lines under λ_t
- $F_1(q,t) = ma(q,t)$ = force field

$$F_1(q,t) = -\frac{\partial V_1}{\partial q}(q,t)$$

Double well , τ = 1.0



Double well , $\tau = 0.1$



$$H(q,p,t) = H_0(q,p;\lambda_t) + V_1(q,t)$$

Next step: simulate quantal evolution under the Hamiltonian

$$\hat{H}(t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + V(q;\lambda_t) + V_1(q,t)$$

Summary:

- Classical shortcuts are *interesting*
- Classical shortcuts are *useful*
- Solutions exist for both strong and weak classical shortcuts

strong - $H_1(q,p,t)$ non-local
weak - $V_1(q,t)$ local

Quantal-classical correspondence

$$\left[\hat{H}_1,\hat{H}_0\right]=i\hbar\dot{\lambda}\left(\nabla\hat{H}_0-diag\left(\nabla\hat{H}_0\right)\right) \qquad \left\langle n\middle|\hat{H}_1\middle|n\right\rangle=0$$

$$\left\{H_1,H_0\right\}=\dot{\lambda}\left(\nabla H_0-\left\langle\nabla H_0\right\rangle\right) \qquad \left\langle H_1\right\rangle=0$$