Experimental Test of Quantum Jarzynski Equality with a Trapped Ion System

2014, Jul. 18

Shortcuts to Adiabaticity, Telluride, CO

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Tsinghua University Center for Quantum Information

Outline

Quantum Jarzynski Equality

- Classical Jarzynski Equality
- Definition of Work in quantum system

Introduction to a Trapped ion system

- Trapping ions
- Basic Operations
- Internal and External Degree of Freedom

Experimental Test of Quantum Jarzynski Equality

- Experimental Procedure
- Thermal State Preparation
- Projective Measurement
- Work Distribution

Conclusion and Outlook

Our Group and Collaborations

Graduate Students

Zhang Xiang, Um Mark, Zhang Junhua, An Shuoming, Wang Ye, Dingshun Lv






Shen Yangchao, Zhang Kuan Lu Yao



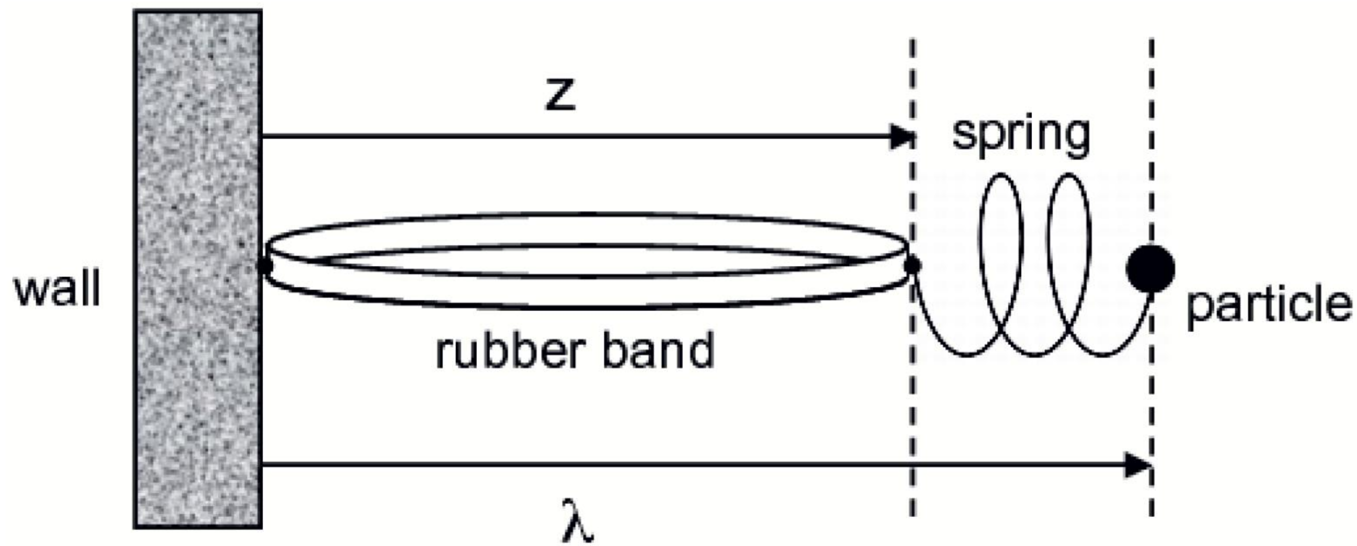
Post-doc.

Dahyun Yum

Theoretical Collaborations

Quan Haitao
Zhang Jingning
Yin Zhangqi
Luming Duan
Shen Chao

Jarzynski Equality



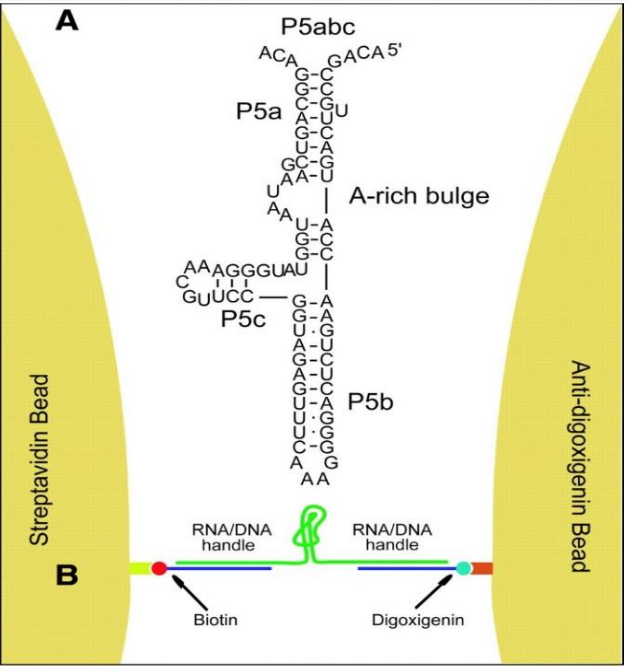
$$\langle W \rangle \geq \Delta F$$

Fluctuation Dissipation Theorem $$\langle W \rangle \approx \Delta F + \sigma^2 / 2k_B T$$

Jarzynski Equality $$\left\langle e^{-\beta W} \right\rangle = e^{-\beta \Delta F}$$

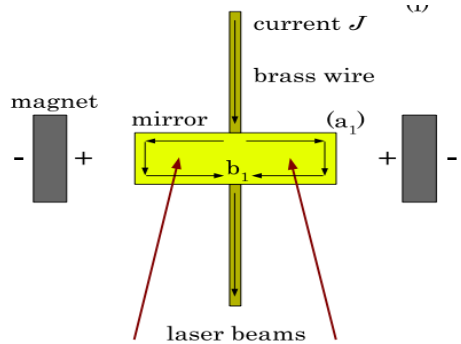
C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997)
Also, G.E. CrooksPhys. Rev. E 60, 2721{2726 (1999).

Experimental Demonstration on Classical Equality



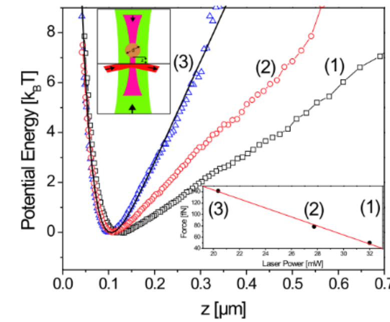
Mechanical Oscillator:

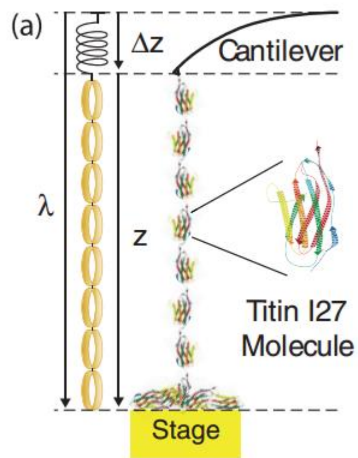
F. Douarche, S. Ciliberto,
A. Petrosyan, and I. Rabbiosi,
Europhys. Lett. 70, 593 (2005)

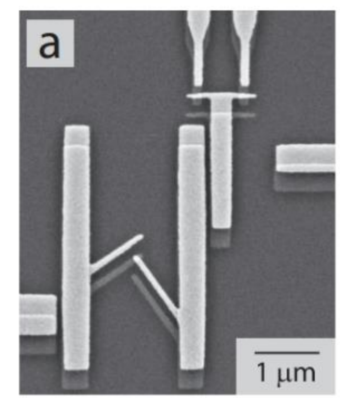


Colloidal Particles:

V. Blickle, T. Speck, L. Helden,
U. Seifert, and C. Bechinger,
Phys. Rev. Lett. 96, 070603 (2006)







RNA:

J. Liphardt, S. Dumont, S. B. Smith, I. J. Tinoco, and C. Bustamante, Science 296, 1832 (2002)

Titin:

N. C. Harris, Y. Song, and C.-H. Kiang,
Phys. Rev. Lett. 99, 068101 (2007)

Electronic System:

O.-P. Saira, Y. Yoon, T. Tanttu, M. Möttönen,
D. V. Averin, and J. P. Pekola,
Phys. Rev. Lett. 109, 180601(2012)

Quantum Jarzynski Equality

S. Mukamel, Phys. Rev. Lett. **90**, 170604 (2003), H. Tasaki, cond-mat/0009244 (2000), J. Kurchan, cond-mat/0007360 (2000).

Work in Quantum Regime $$W = E_m(t_f) - E_n(t_i)$$

Proof of the Equality $$\langle \exp(-\beta W) \rangle = \exp[-\beta(F_f - F_i)] = \frac{Z_f}{Z_i}$$

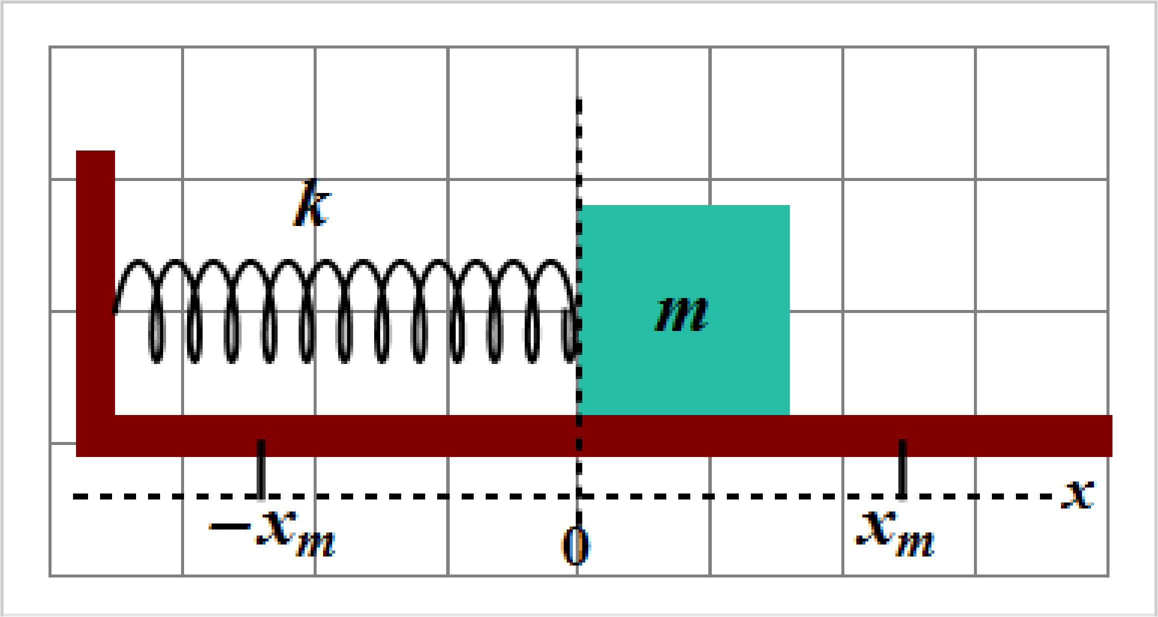
Initial and final free energy $$Z_{i,(f)} = \sum_n \exp[-\beta E_n(t_{i,(f)})] = \exp(-\beta F_{i(f)})$$

$$\langle \exp(-\beta W) \rangle = \frac{1}{Z_i} \sum_n \exp[-\beta E_n(t_i)] \sum_m K_{mn} \exp\{-\beta [E_m(t_f) - E_n(t_i)]\}$$

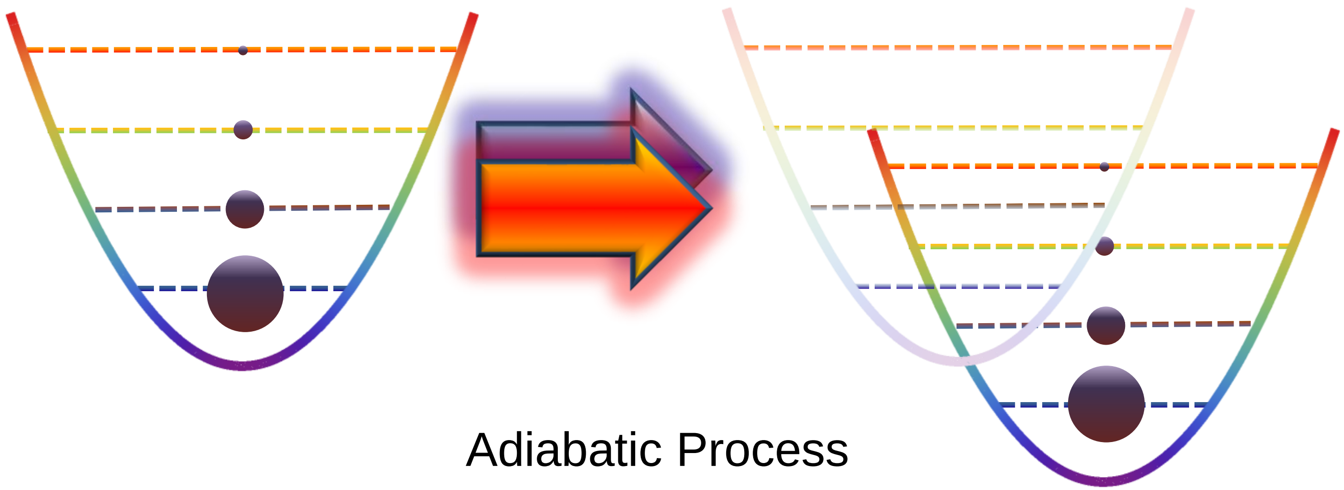
$$= \frac{1}{Z_i} \sum_{nm} K_{mn} \exp[-\beta E_m(t_f)] = \frac{Z_f}{Z_i} \frac{1}{Z_f} \sum_m \exp[-\beta E_m(t_f)] = \frac{Z_f}{Z_i}$$

Quantum Jarzynski Equality

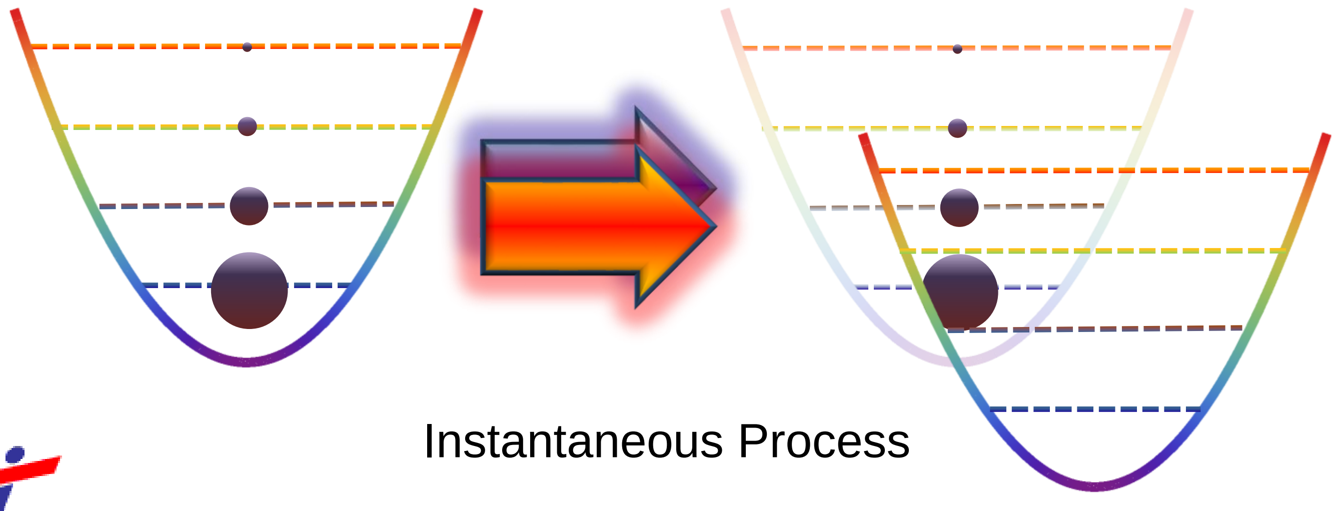
Employing Trapped Cold Ions to Verify the Quantum Jarzynski Equality, Gerhard Huber, Ferdinand Schmidt-Kaler, Sebastian Deffner and Eric Lutz, Phys. Rev. Lett. 070403 (2008).



Quantum Jarzynski Equality



Adiabatic Process



Instantaneous Process

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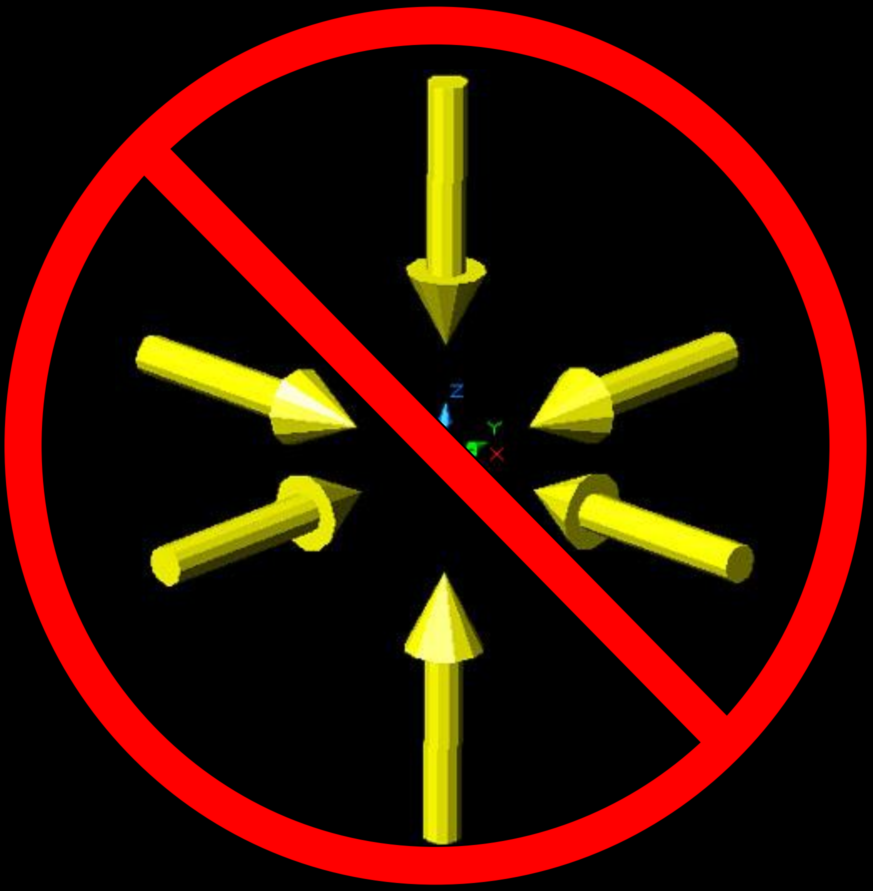
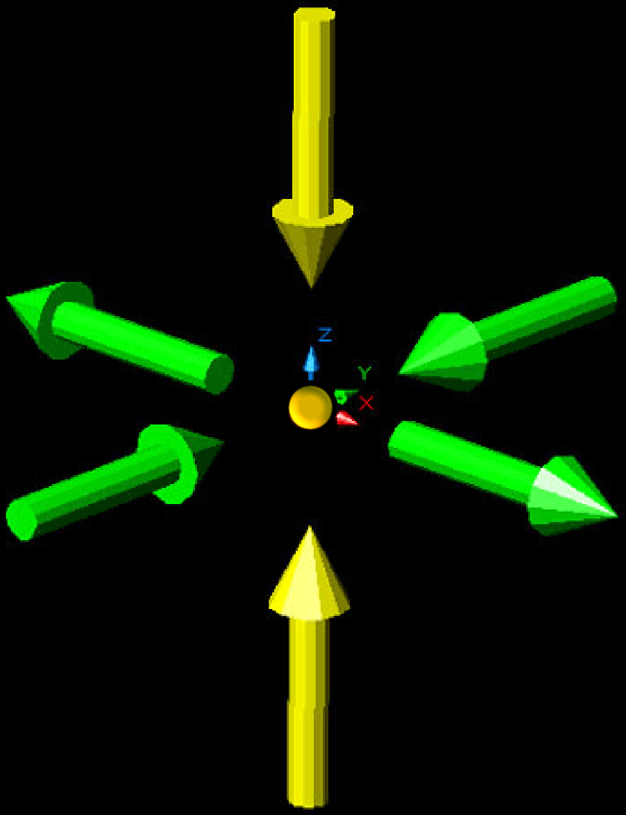
Ion Trap

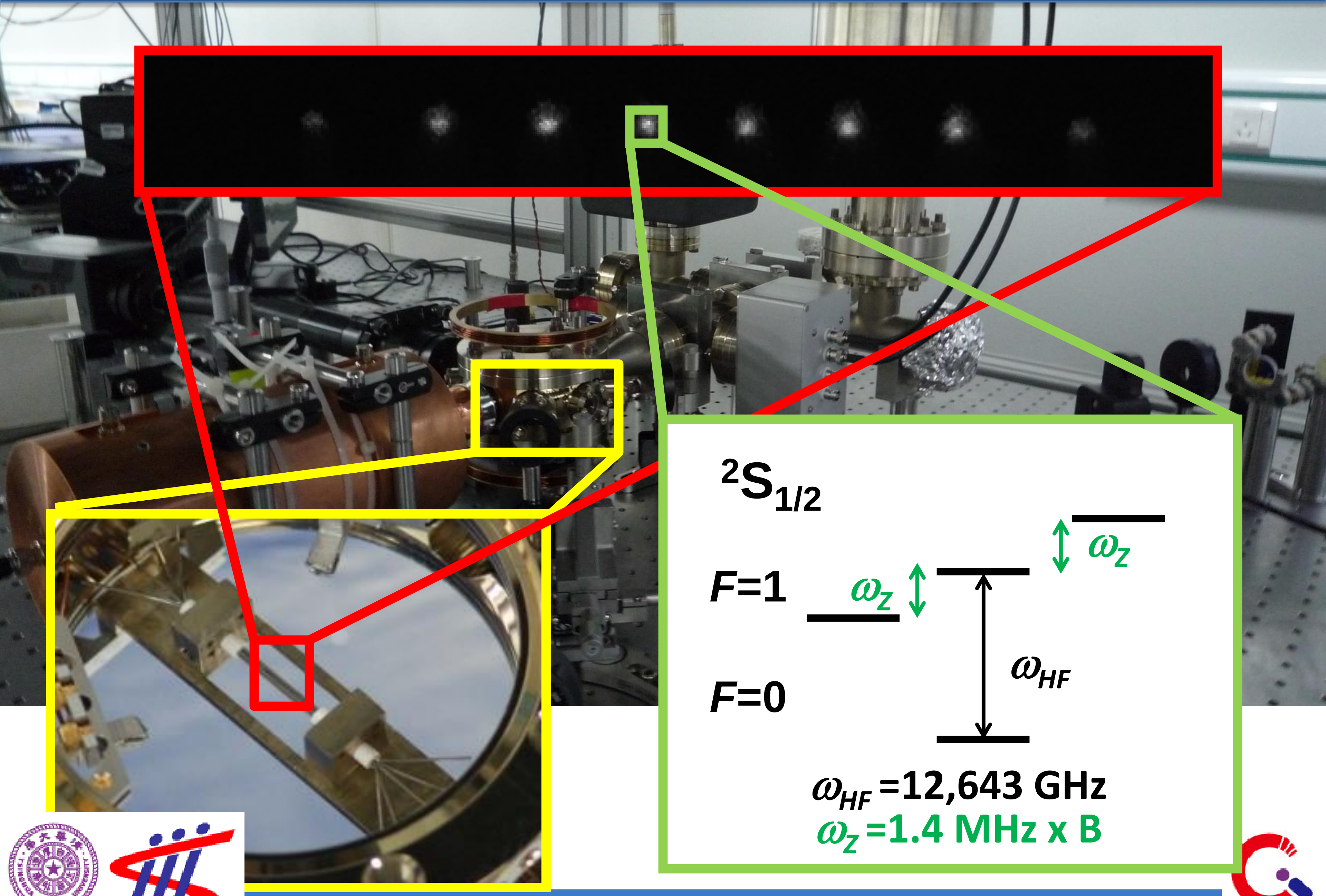
Electric Field Vectors

NO! $\nabla \cdot \mathbf{E} = 0$

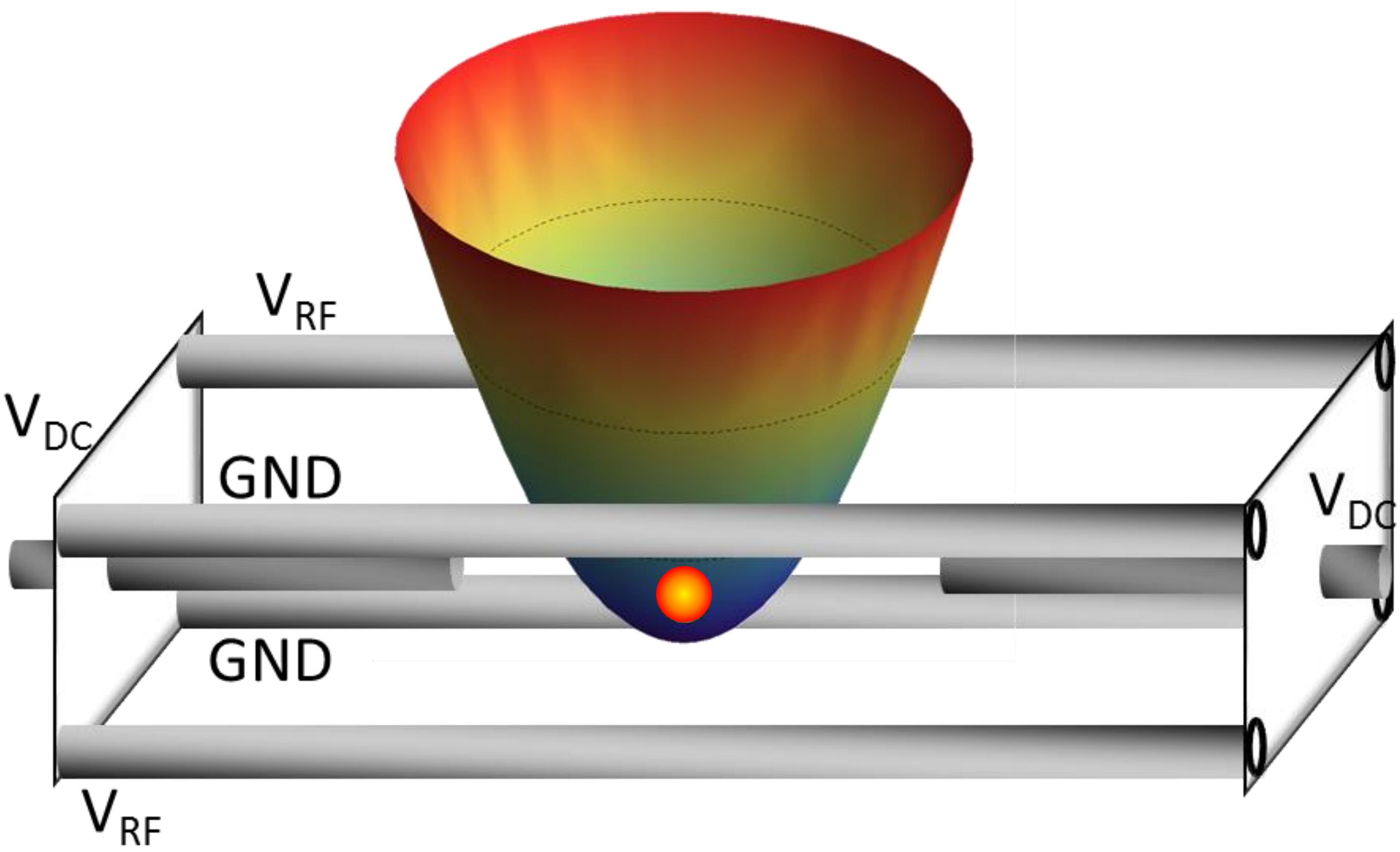




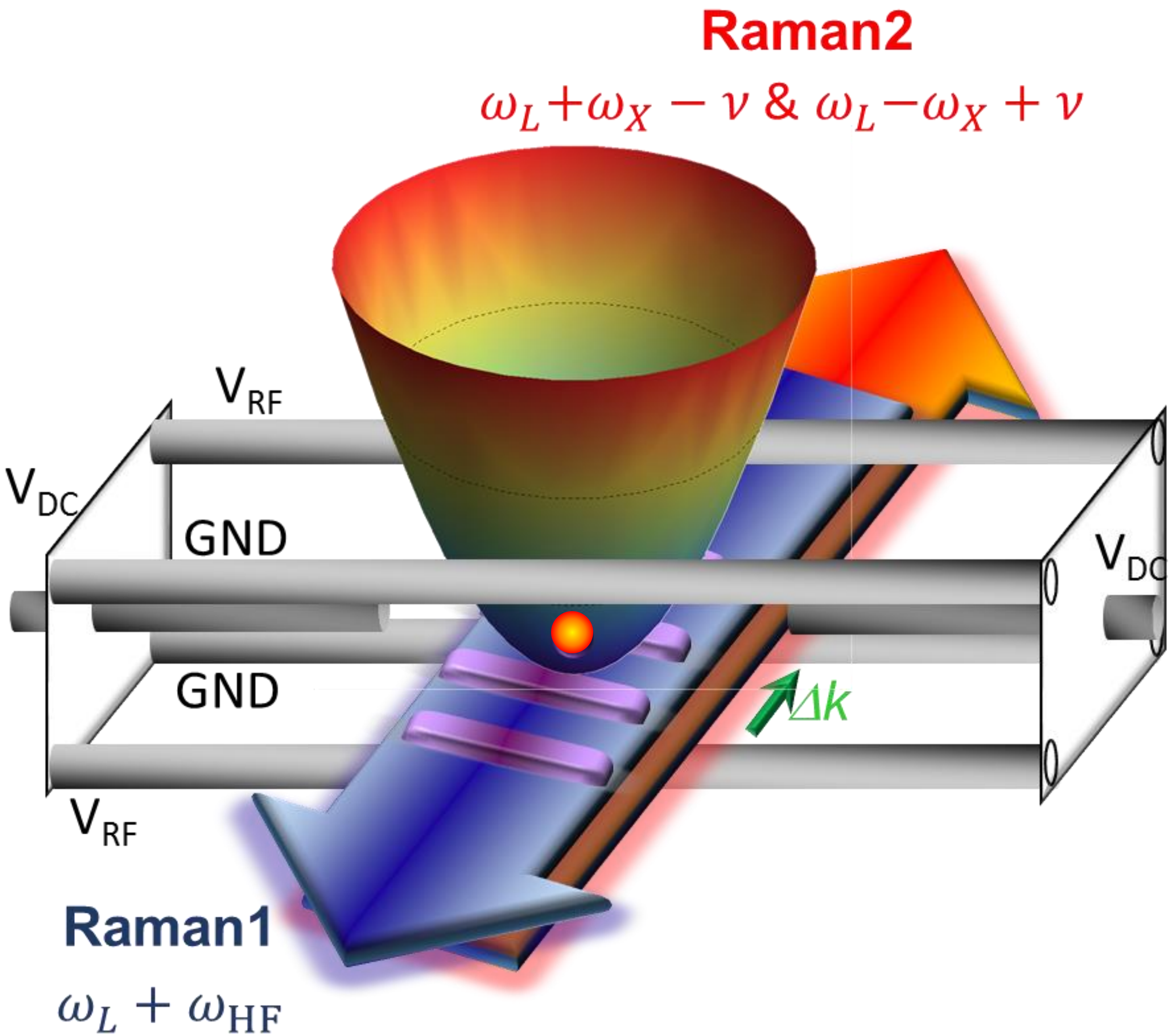
Linear Ion Trap @ Tsinghua University



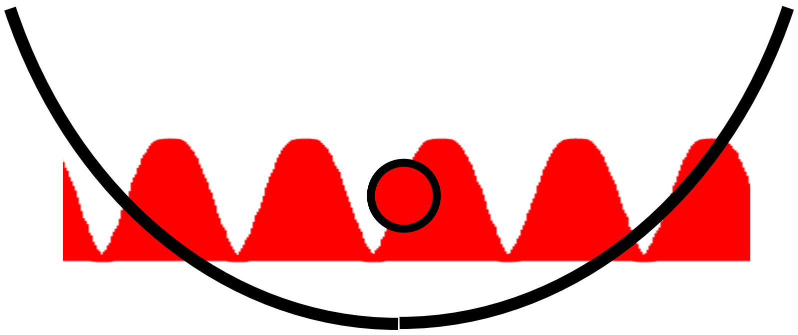
Trapped Ion Harmonic Oscillator



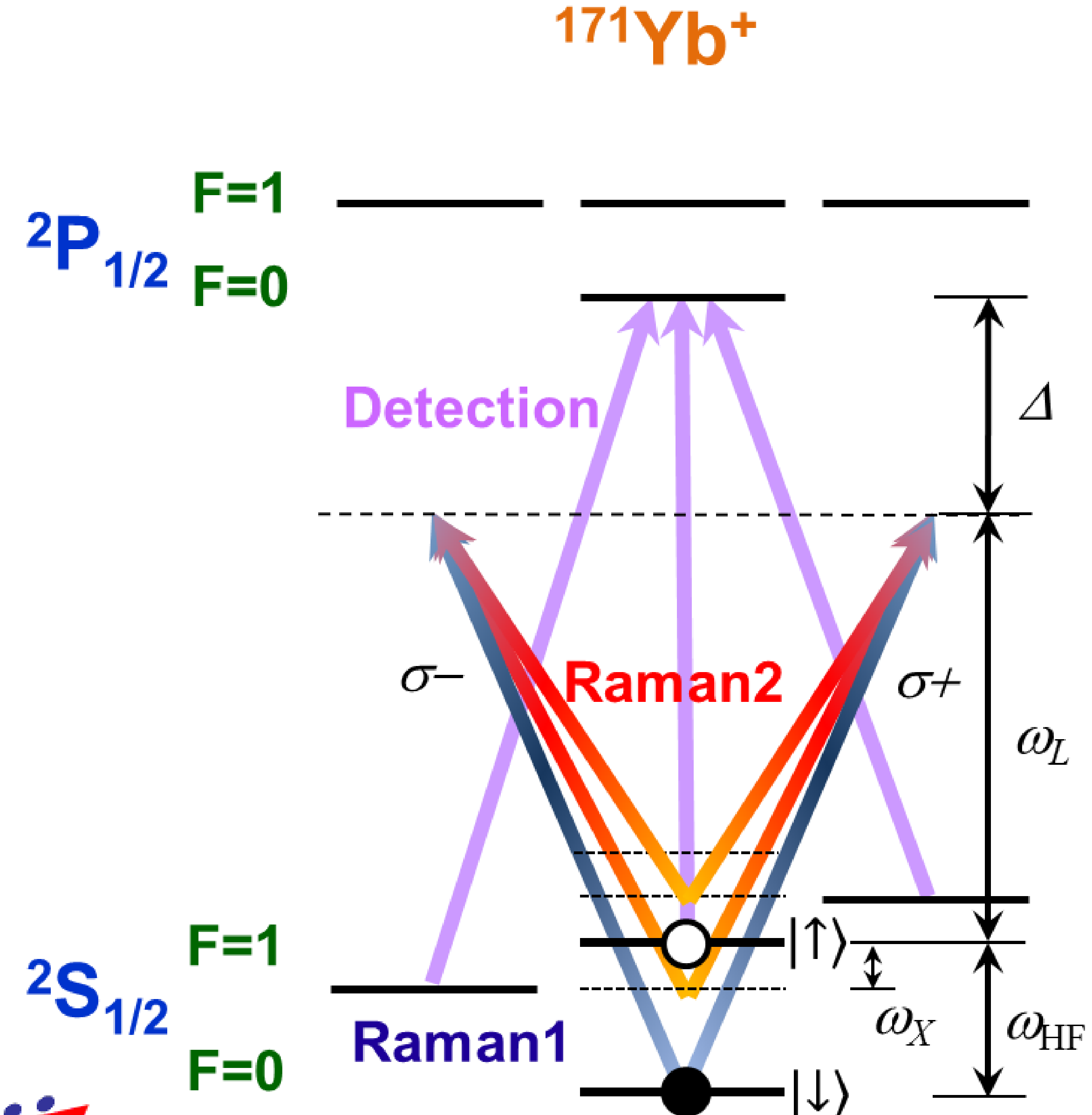
Pulsed Laser Configuration



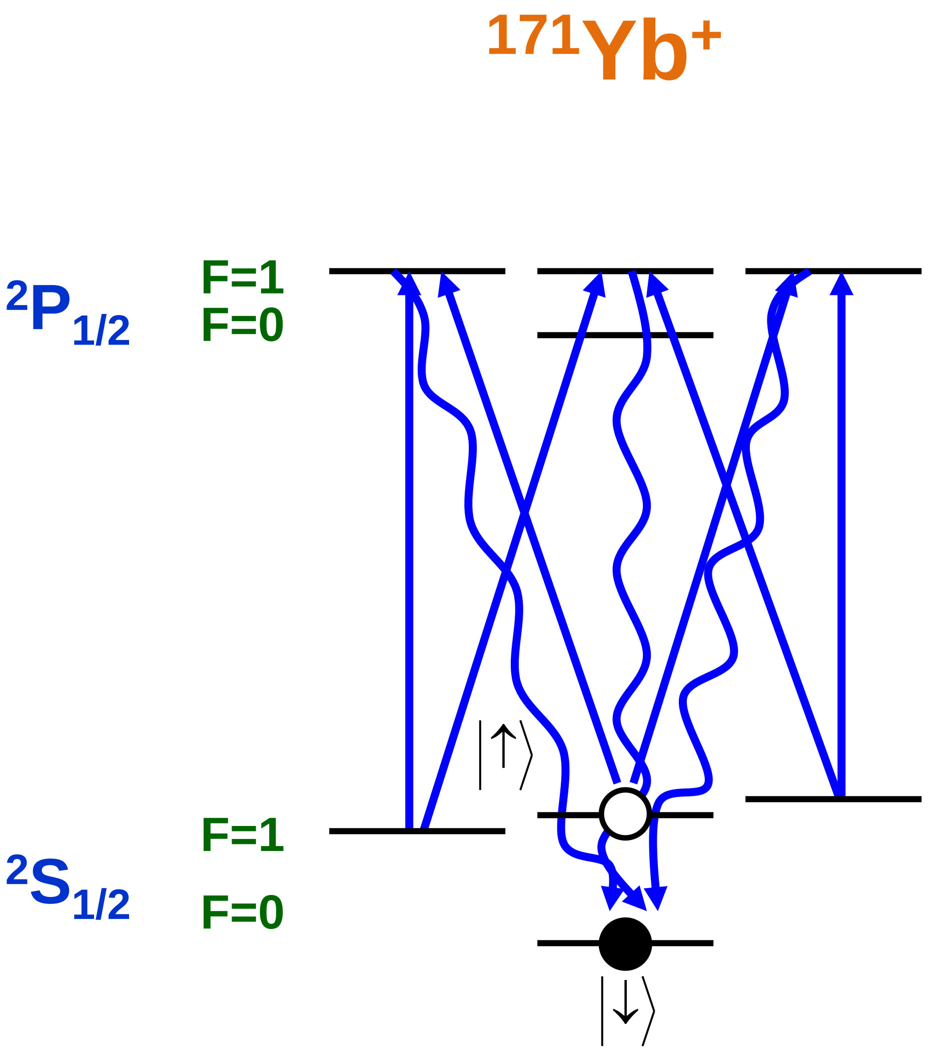
Effective Force induced by Laser beam



Pulsed Laser Configuration



Initialization – optical pumping

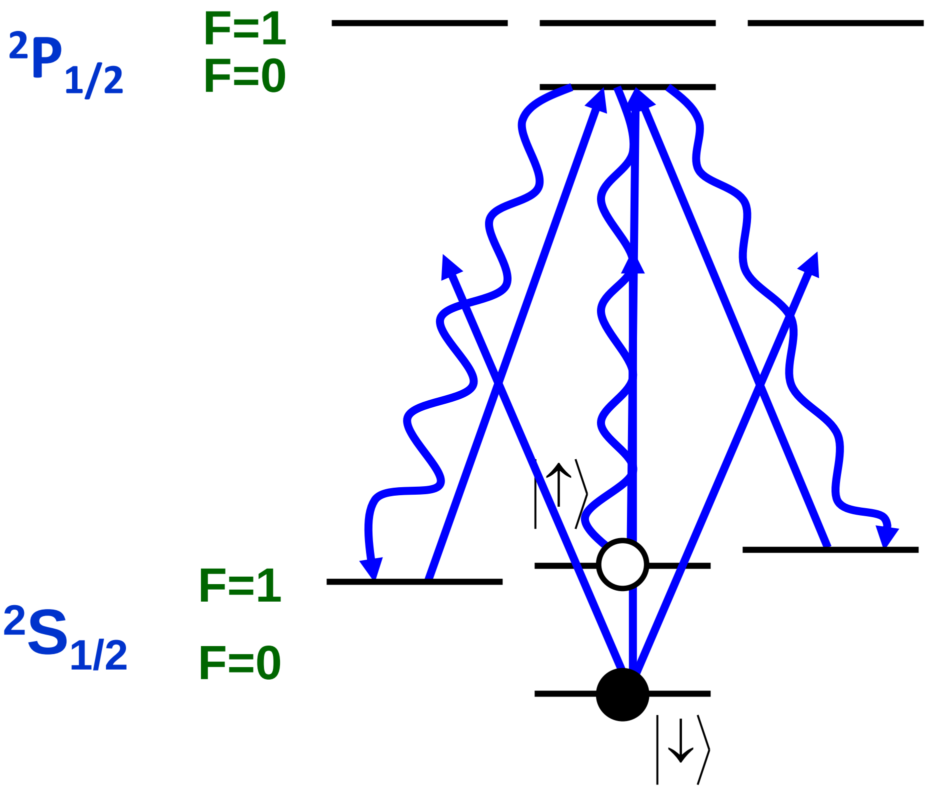


- Duration ~ 1 μs
- Efficiency ~ 99.5%

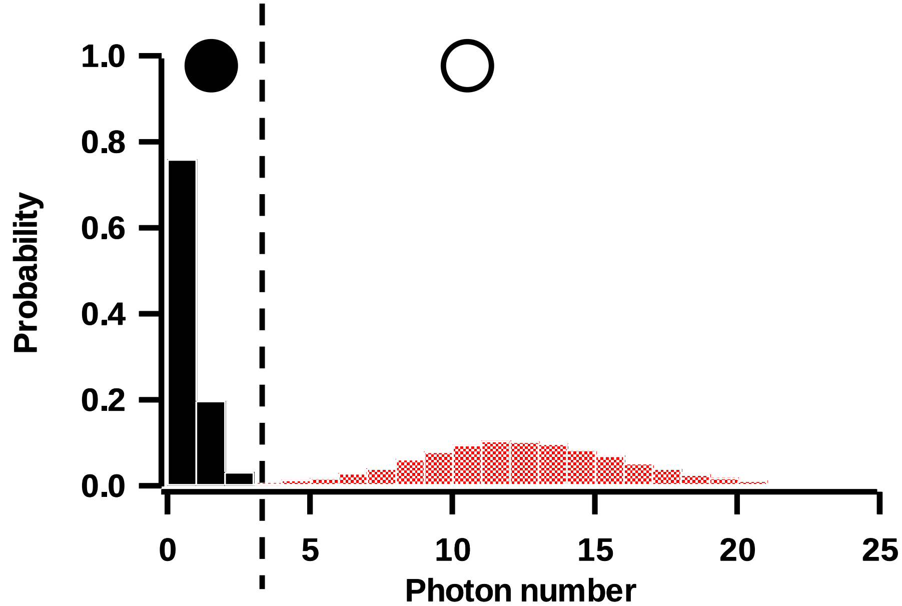
S. Olmschenk, et al., PRA **76**, 052314 (2007)

Detections

$^{171}Yb^+$



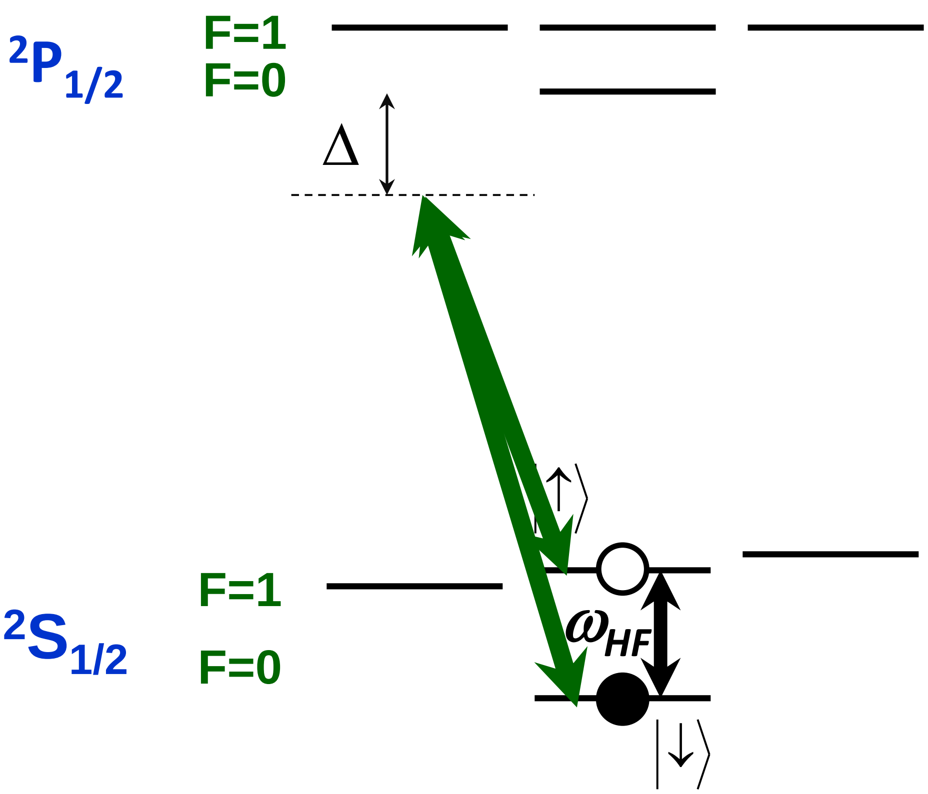
- Duration ~ 0.4 ms
- Efficiency ~ 98.5%

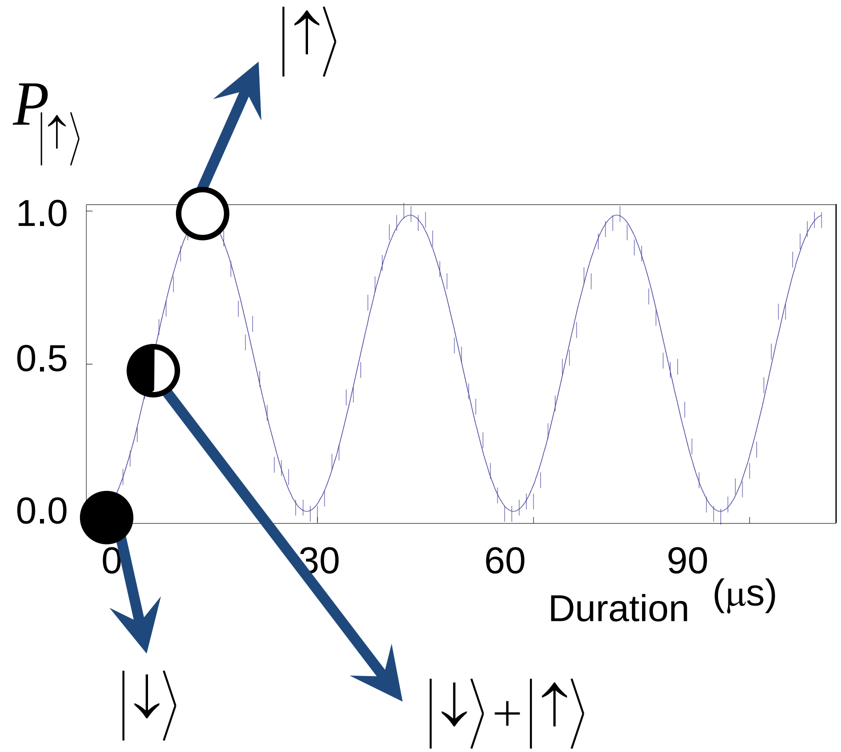


S. Olmschenk, et al., PRA **76**, 052314 (2007)

Operations

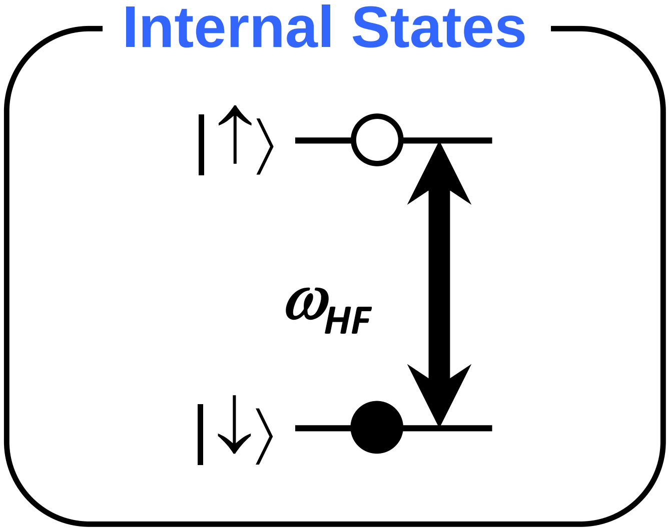
$^{171}Yb^{+}$



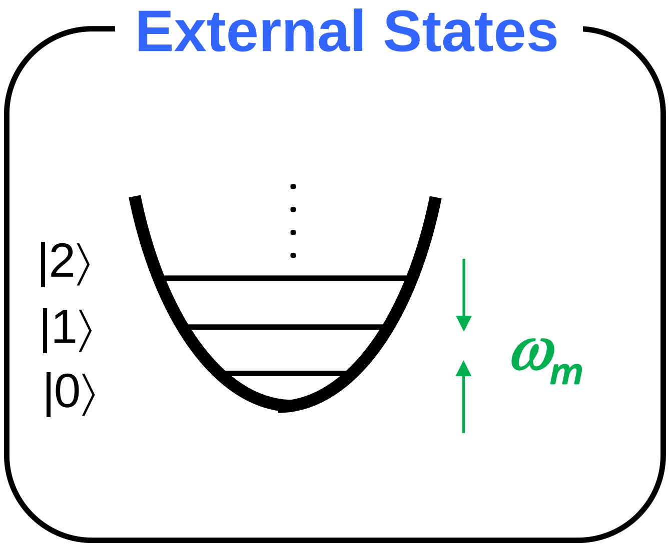


$$H_{carrier} = \hbar\Omega\sigma_x$$

Internal and External Degree of Freedom



$\otimes$



$$H^{(e)} = \frac{\hbar\omega_{HF}}{2}\left(|\downarrow\rangle\langle\downarrow| - |\uparrow\rangle\langle\uparrow|\right)$$

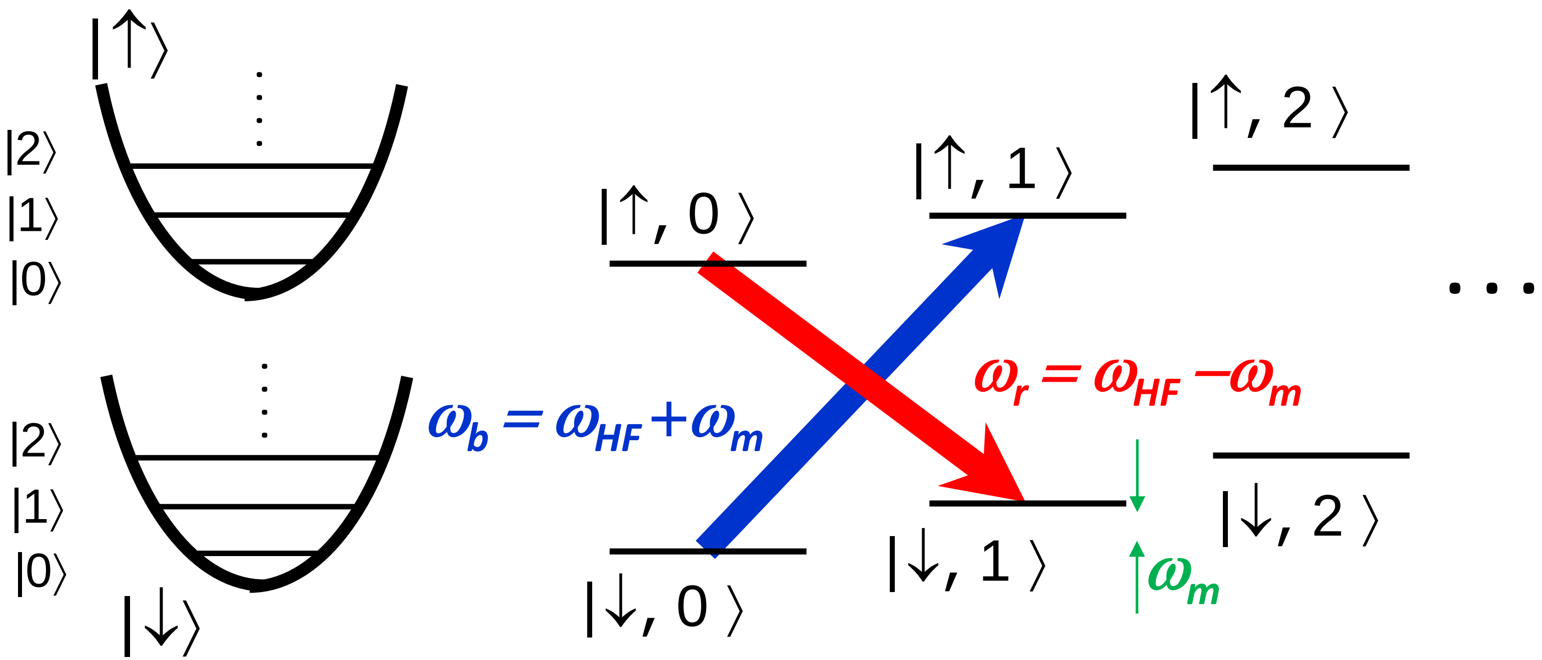
$$= \frac{\hbar\omega_{HF}}{2}\sigma_z$$

$$H^{(m)} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2$$

$$= \hbar\omega_m\left(a^+a + \frac{1}{2}\right)$$

D. Leibfried, et al., RMP 75, 281{324 (2003).

Connecting Internal and External Degree of Feedom



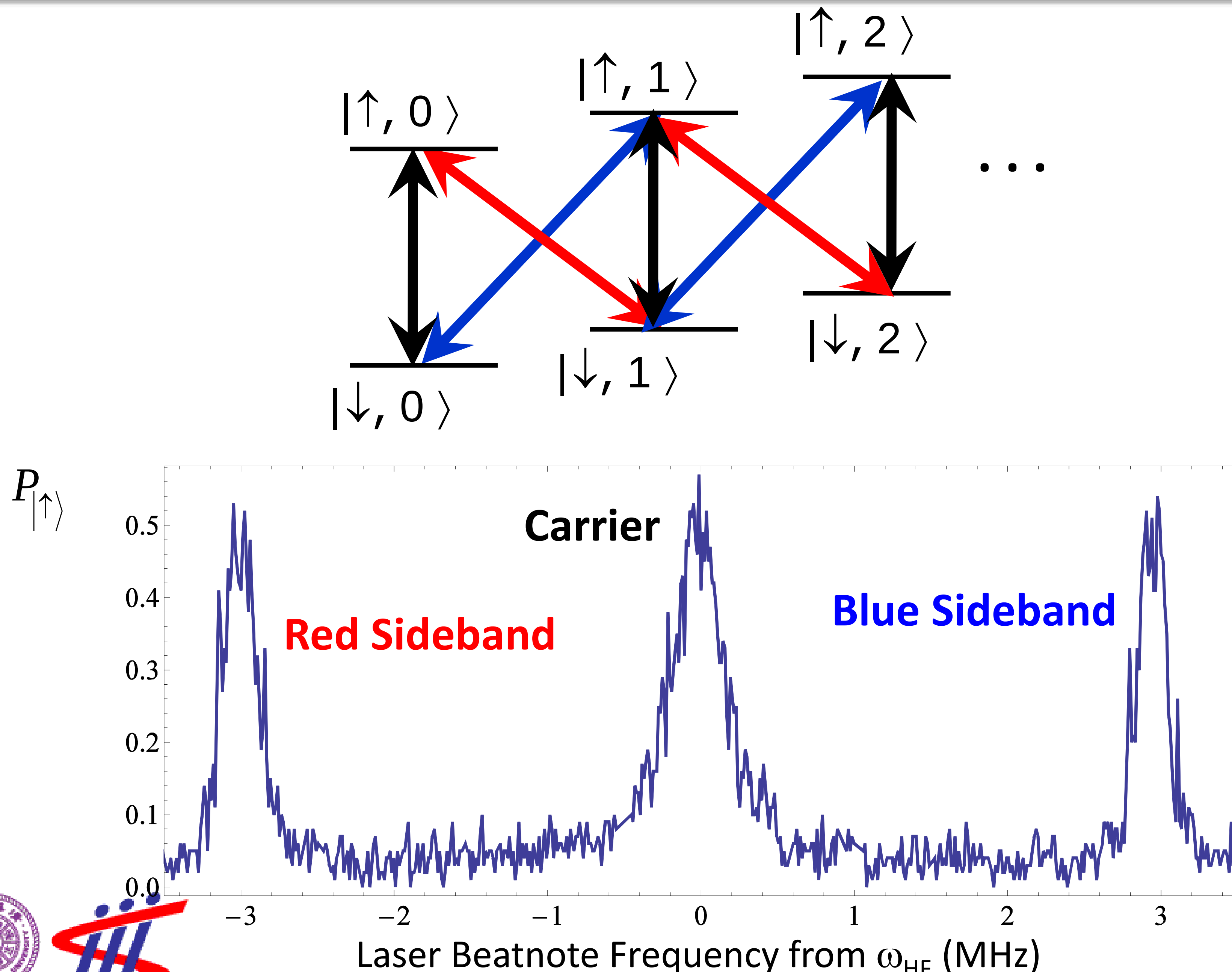
$$H^{(e)} \otimes H^{(m)}$$

$$H_{bsb} = -i\hbar\eta\Omega\sigma^{+}a^{\dagger} + h.c.$$

$$H_{rsb} = -i\hbar\eta\Omega\sigma^{-}a^{\dagger} + h.c.$$

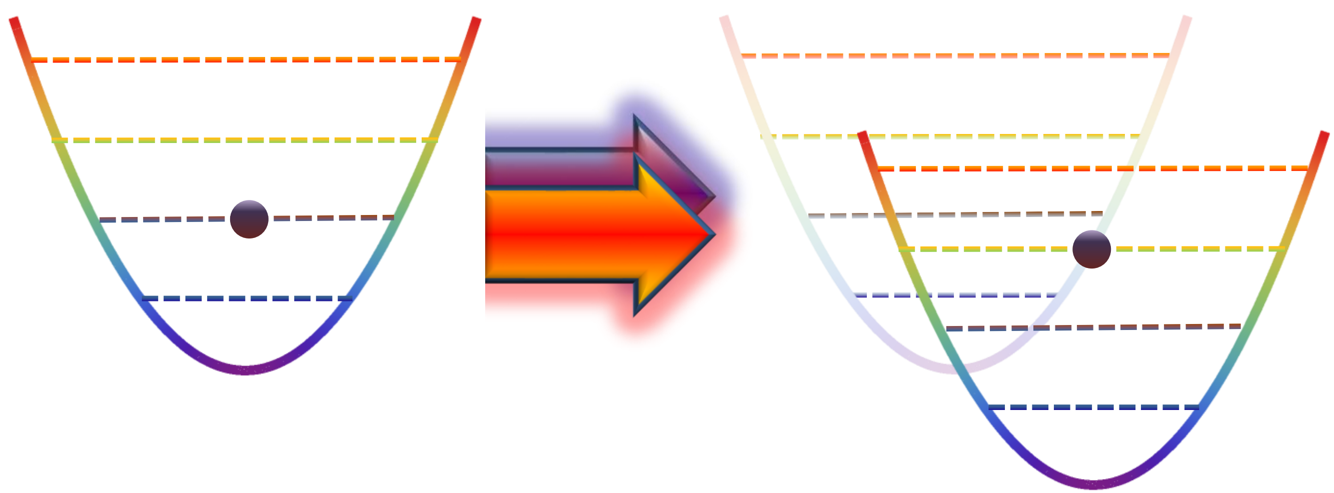
D. Leibfried, et al., RMP 75, 281{324 (2003).

Internal and External Degree of Freedom



Work Distribution

$$P(W) = \sum_{n,\bar{n}} \delta[W - (E_{\bar{n}}(\tau) - E_n(0))]P_{\bar{n}\leftarrow n}P_n^{th}$$



Proposal for the Test with Trapped Ion System

Employing Trapped Cold Ions to Verify the Quantum Jarzynski Equality, Gerhard Huber, Ferdinand Schmidt-Kaler, Sebastian Deffner and Eric Lutz, Phys. Rev. Lett. 070403 (2008).

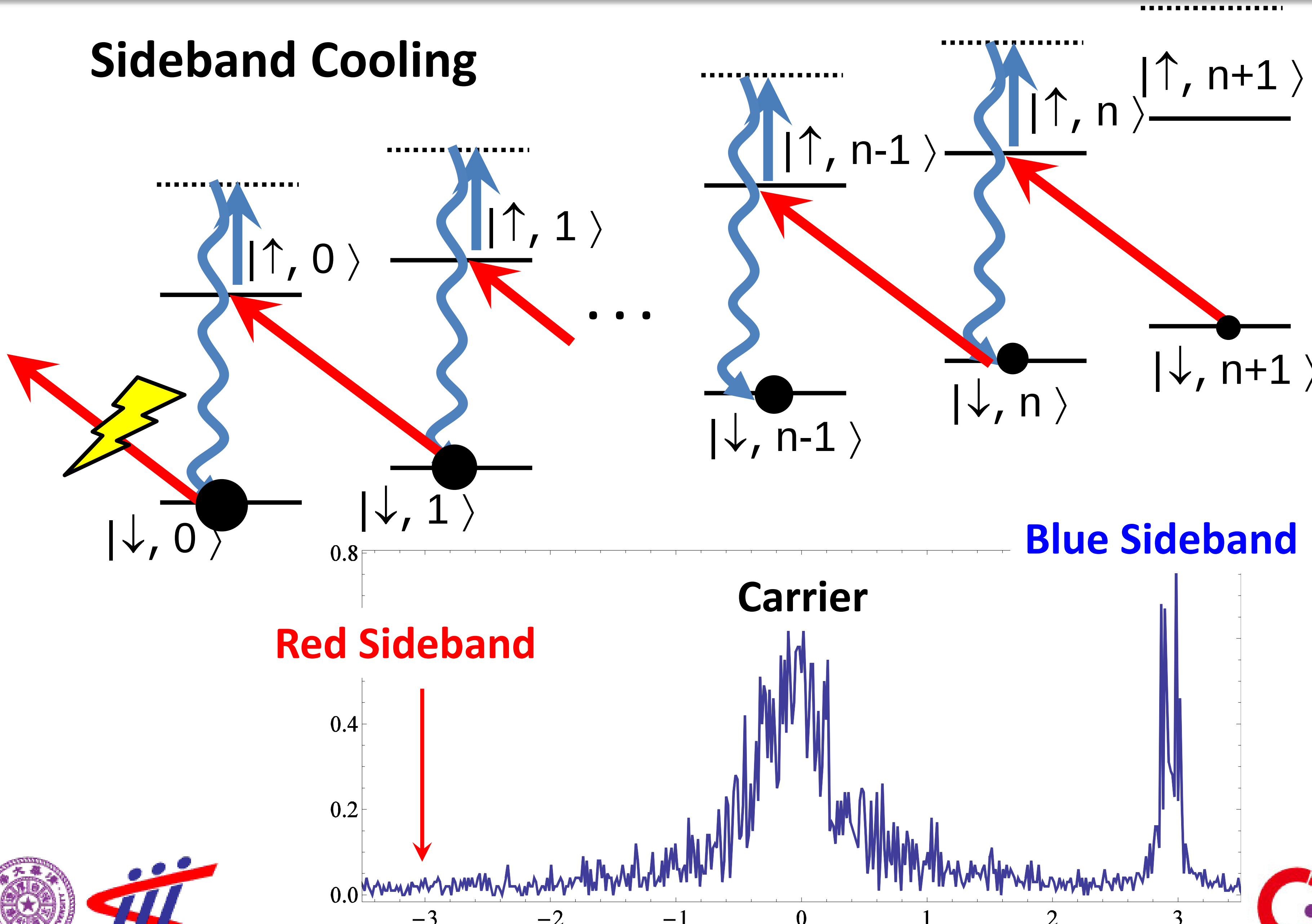
1. Prepare Thermal State
2. Project to a phonon number state, n
3. Provide Work on the System
4. Project to a phonon number state, m
5. Repeat the whole sequence from step 1

1. Prepare Thermal State

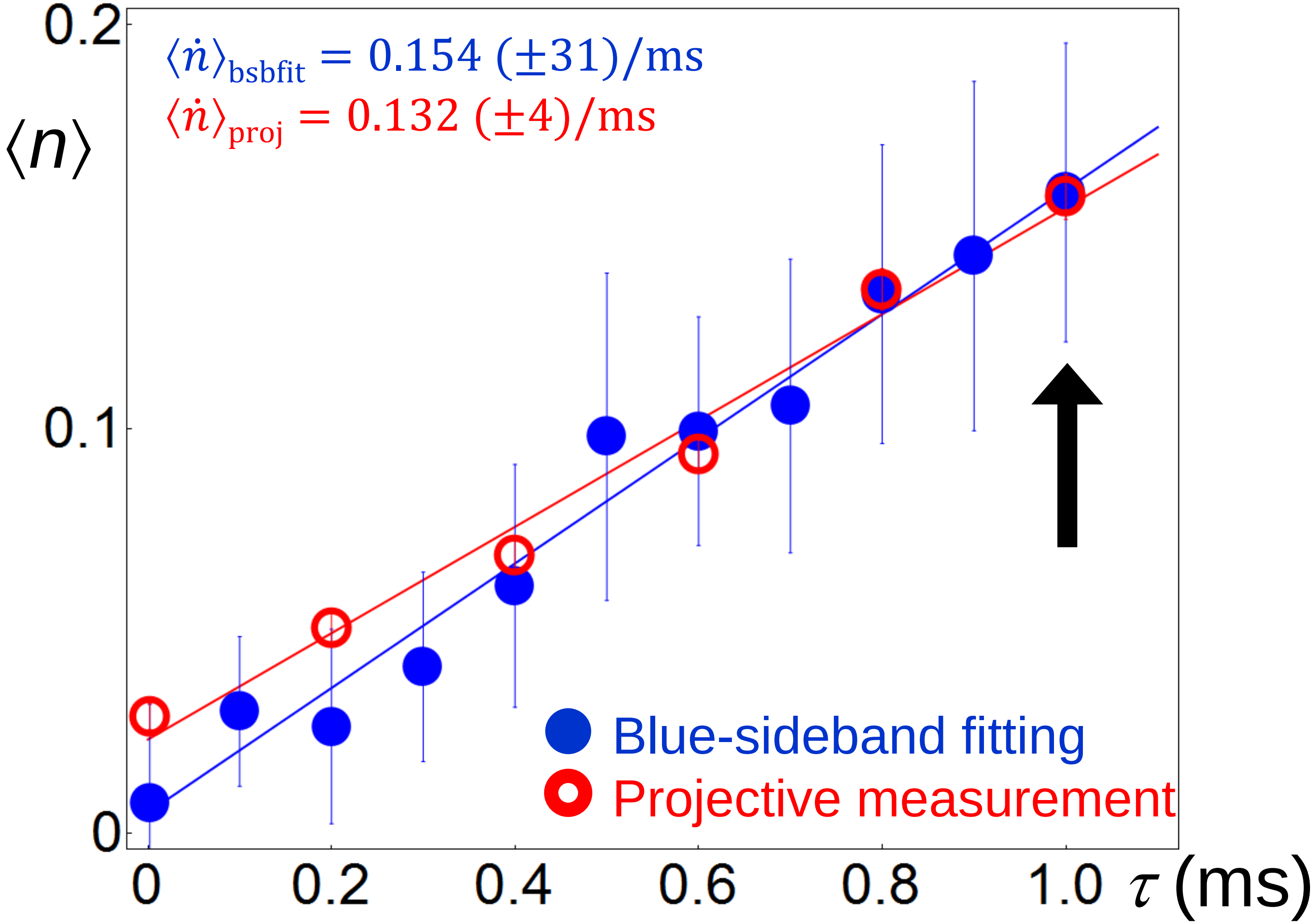
a. Prepare $|n=0\rangle$, motional ground state

b. Let it heat up

a. Prepare |n=0⟩, motional ground state



b. Heating Mechanism

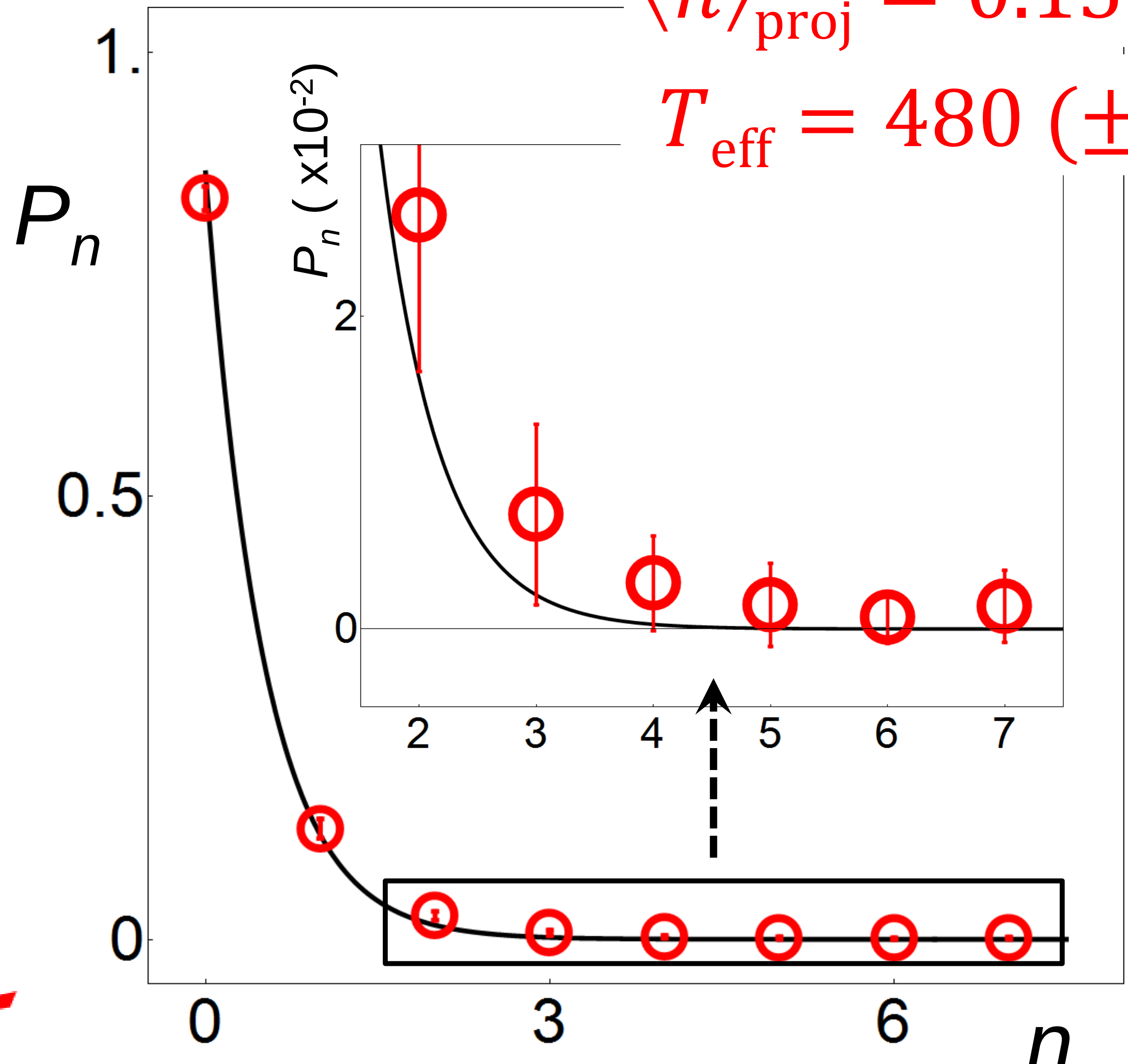


Q. A. Turchette et al., Phys. Rev. A. **61,** 063418 (2000).
Q. A. Turchette et al., Phys. Rev. A. **62,** 053807 (2000).
C. J. Myatt, et al., Nature 403, 269{273 (2000).

Thermal State Detection – Projective measurement

$\langle n \rangle_{\text{proj}} = 0.157(\pm 6)$

$T_{\text{eff}} = 480\ (\pm 8)\text{nK}$



2. Project to a phonon number state, |n⟩

1. Prepare Thermal State
2. Project to a phonon number state, n
3. Provide Work on the System
4. Project to a phonon number state, m
5. Repeat the whole sequence from step 1

$|\downarrow,n\rangle$, $m_i=0$

$|\downarrow,n-1\rangle$, $m_i=m_i+1$

Phonon Subtraction

1. Carrier

$|\downarrow,n\rangle \rightarrow |\uparrow,n\rangle$

2. Adiabatic BSB

If $n\neq0$, $|\uparrow,n\rangle \rightarrow |\downarrow,n-1\rangle$

If $n=0$, $|\uparrow,0\rangle \rightarrow |\uparrow,0\rangle$

Detection: Fluorescence?

If $|\downarrow,n-1\rangle$ no fluorescence

If $|\uparrow,0\rangle$ fluorescence

No

Yes

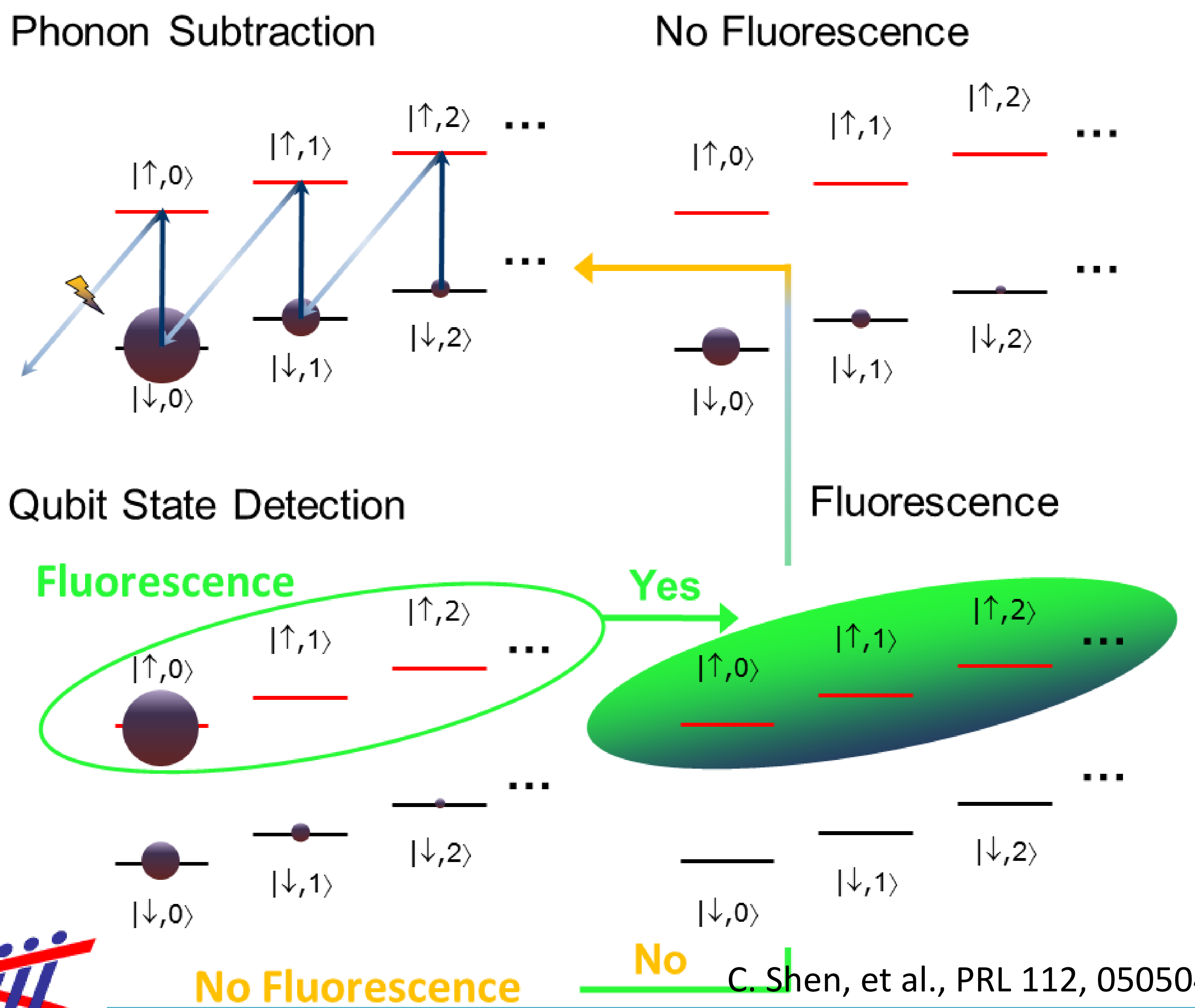
$m_i=n+1$

preparation of $|\downarrow,n\rangle$ state

- m_i: number of iterations

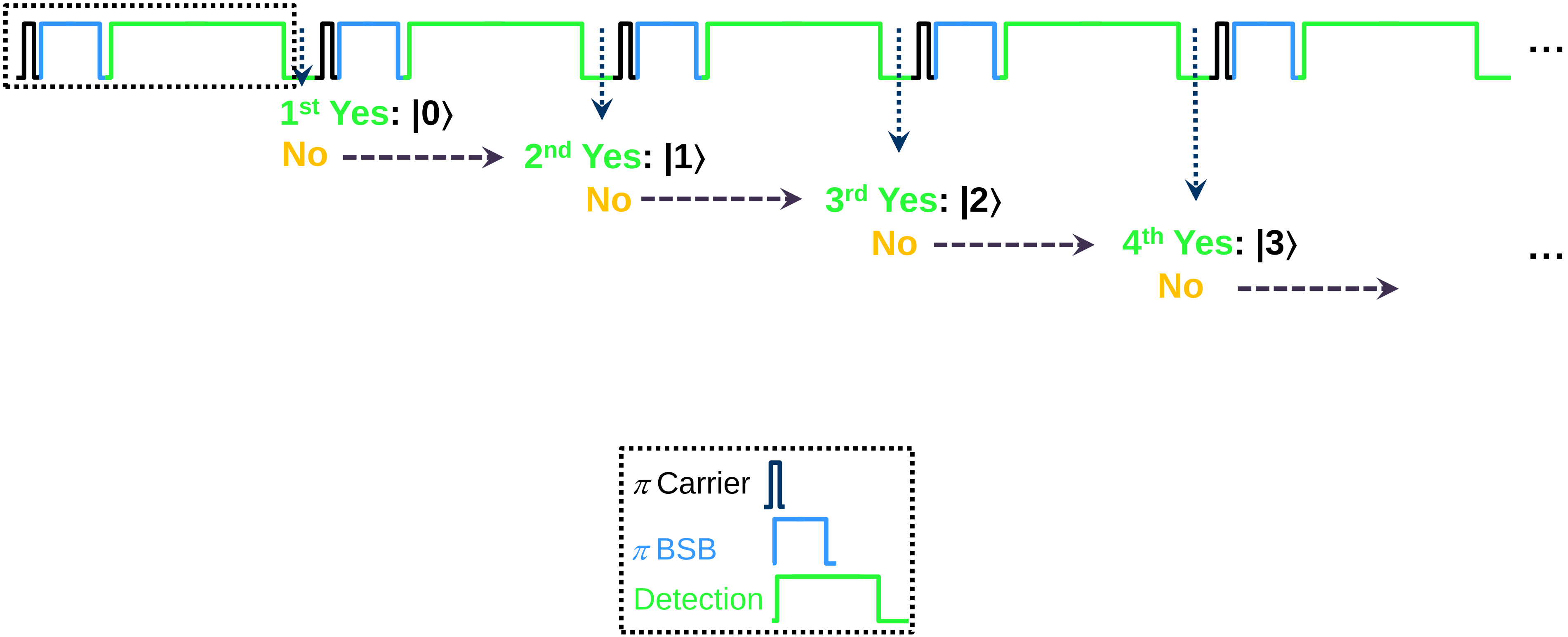
C. Shen, et al., PRL 112, 050504 (2014).

Detection of Phonon State by Projective Measurement



C. Shen, et al., PRL 112, 050504 (2014).

Experimental Procedure

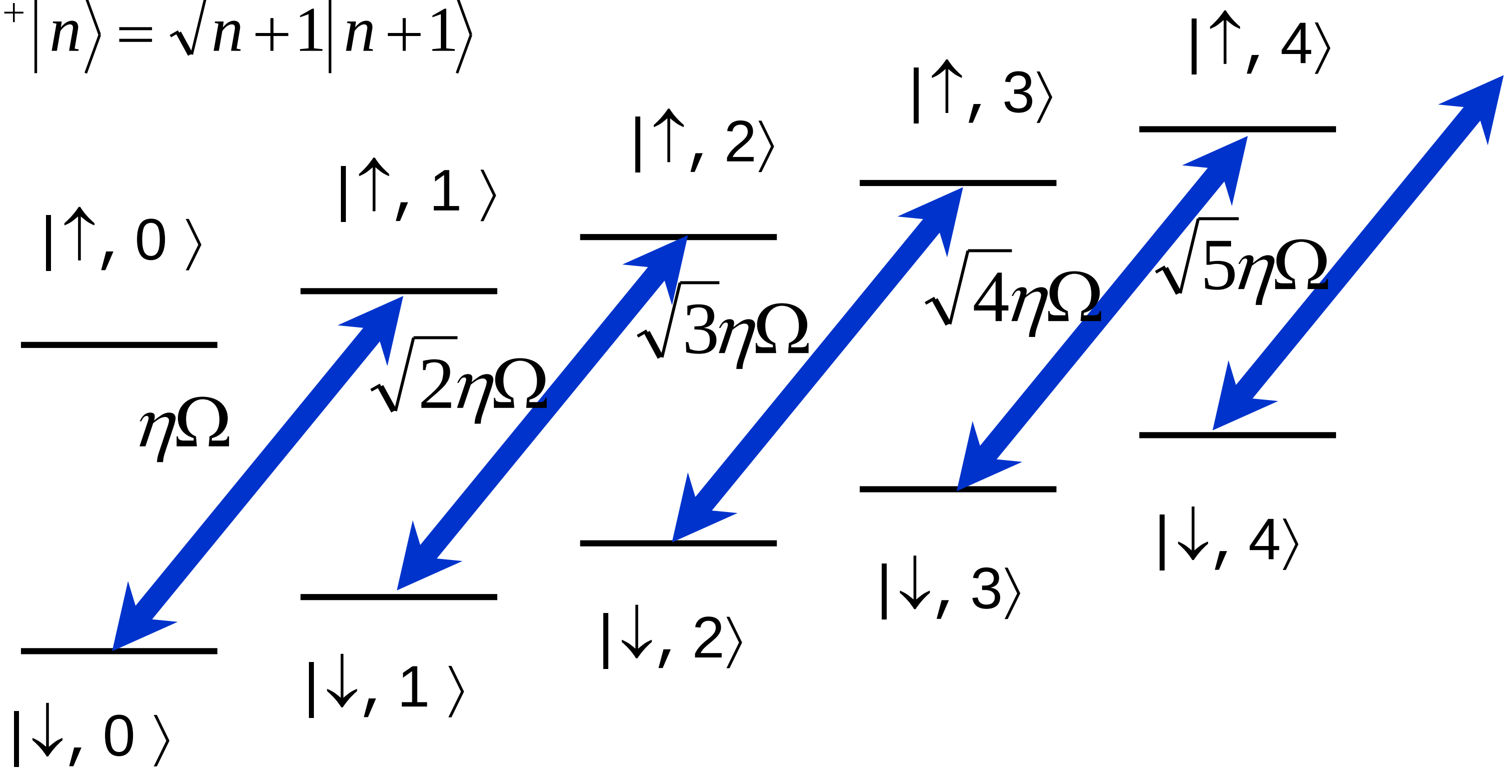


Ion-Motion Coupling: Blue Sideband Transition

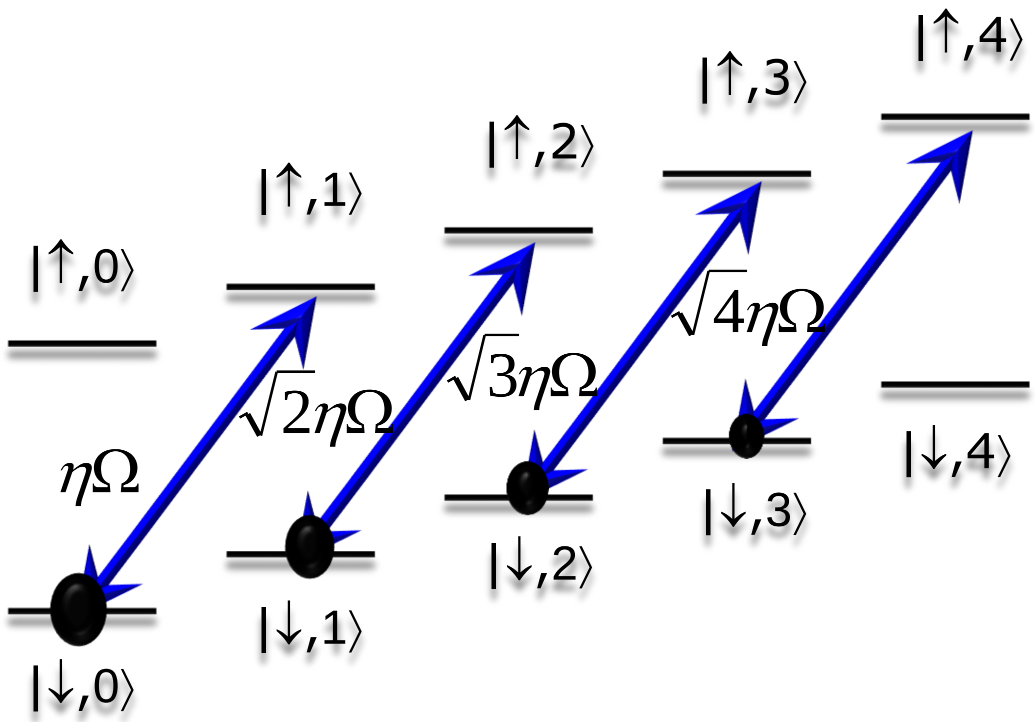
"Blue Sideband"

$$H_{rsb} = (\hbar/2)\eta\Omega\left[\hat{\sigma}_+ a^+ e^{i\varphi} + \hat{\sigma}\_a e^{-i\varphi}\right]$$

$$a^+|n\rangle = \sqrt{n+1}|n+1\rangle$$

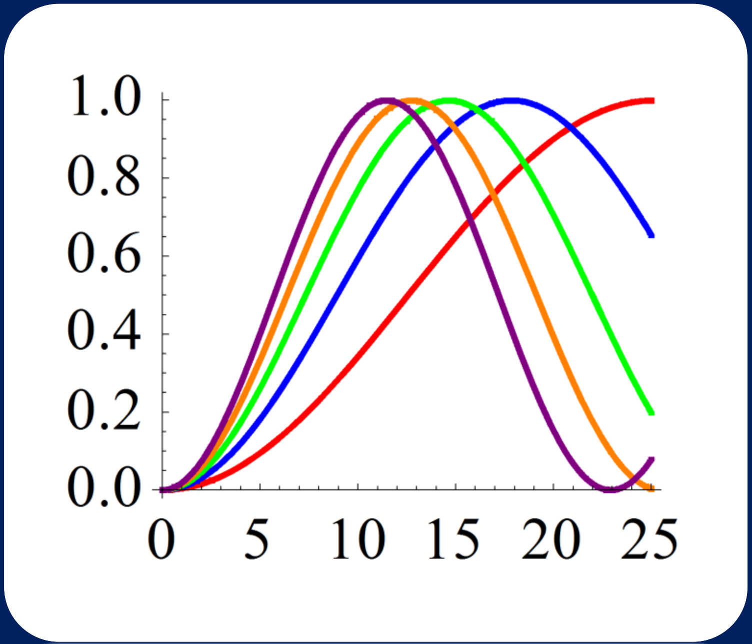


Main Problem

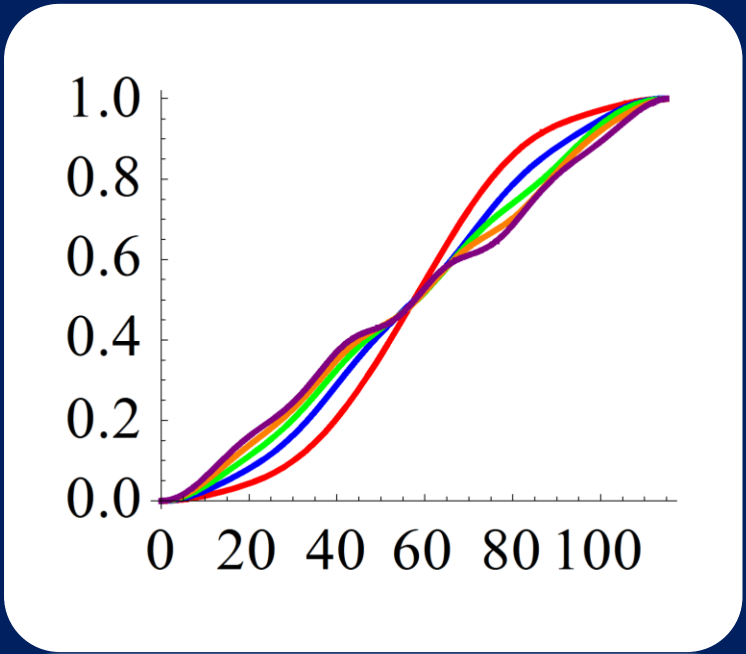


Photon Shift Operation

Blue Sideband

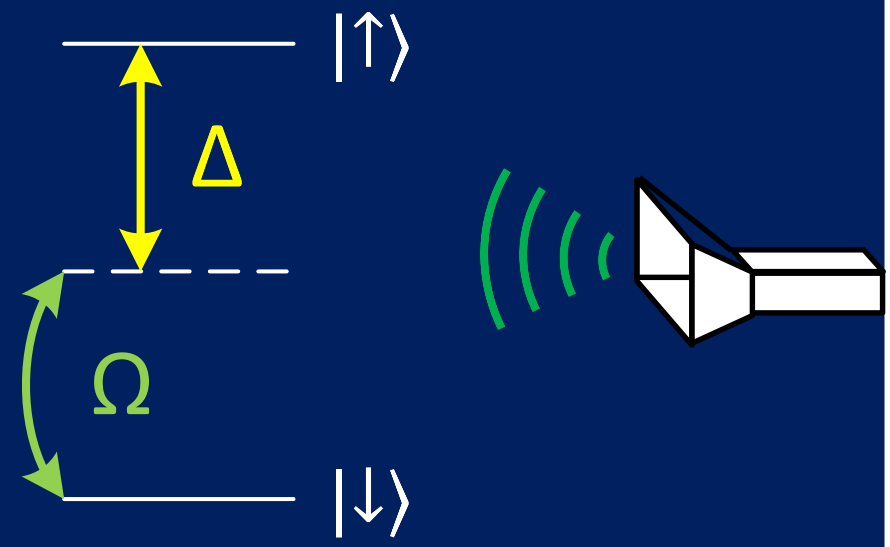


Adiabatic Passage



Rapid Adiabatic Passage

Qubit system in $^{171}\mathrm{Yb}^+$



Interaction Picture:

$$H_{GLZ}(t) = \frac{\hbar}{2}\boldsymbol{\sigma}\cdot\boldsymbol{B}(t)$$

$$B_z = \Delta(t)$$

$$B_x = \Omega(t)\cos\varphi(t)$$

$$B_y = \Omega(t)\sin\varphi(t)$$

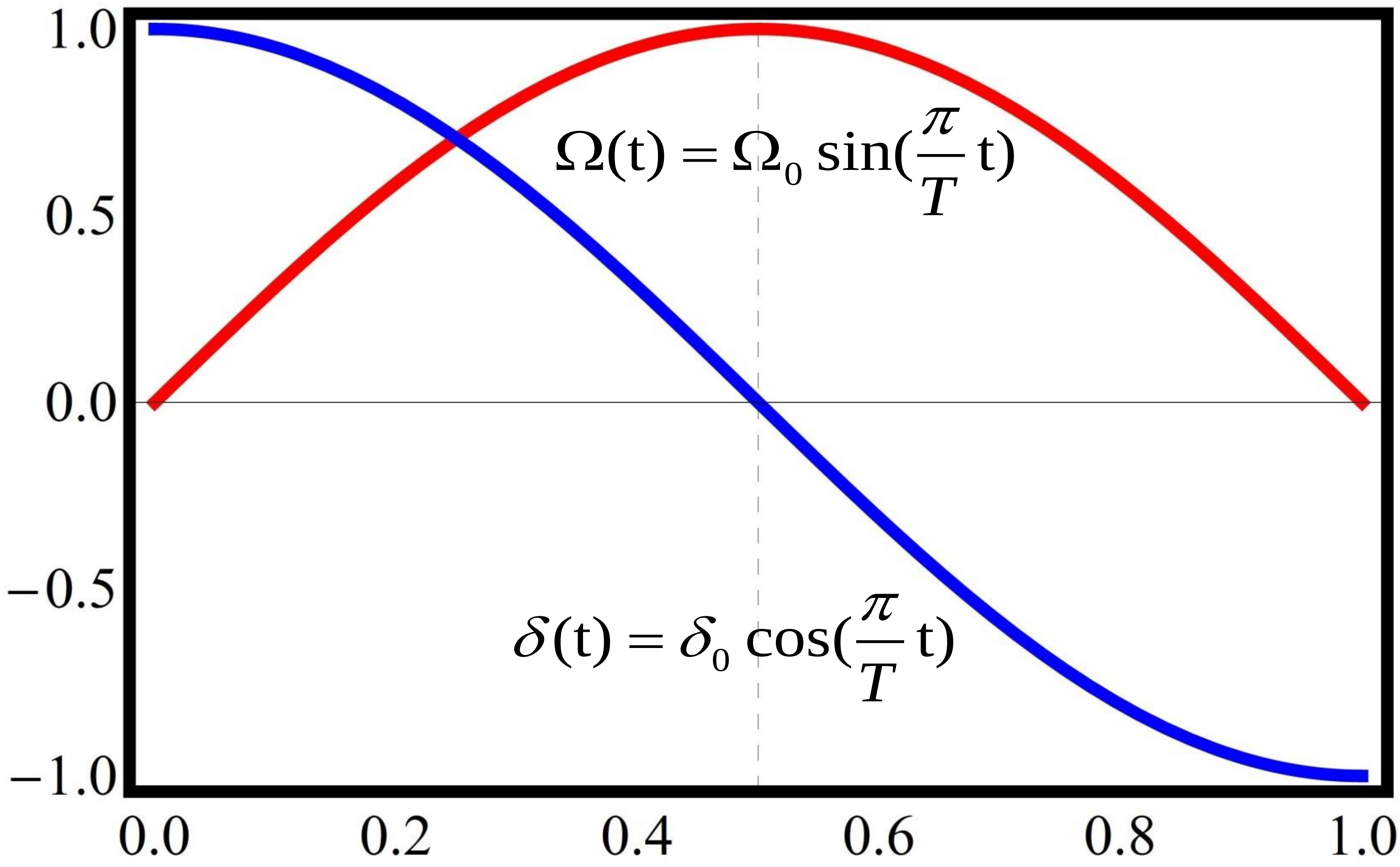
$\Delta(t)$ ~ Detuning

$\Omega(t)$ ~ Amplitude

$\varphi(t)$ ~ Phase

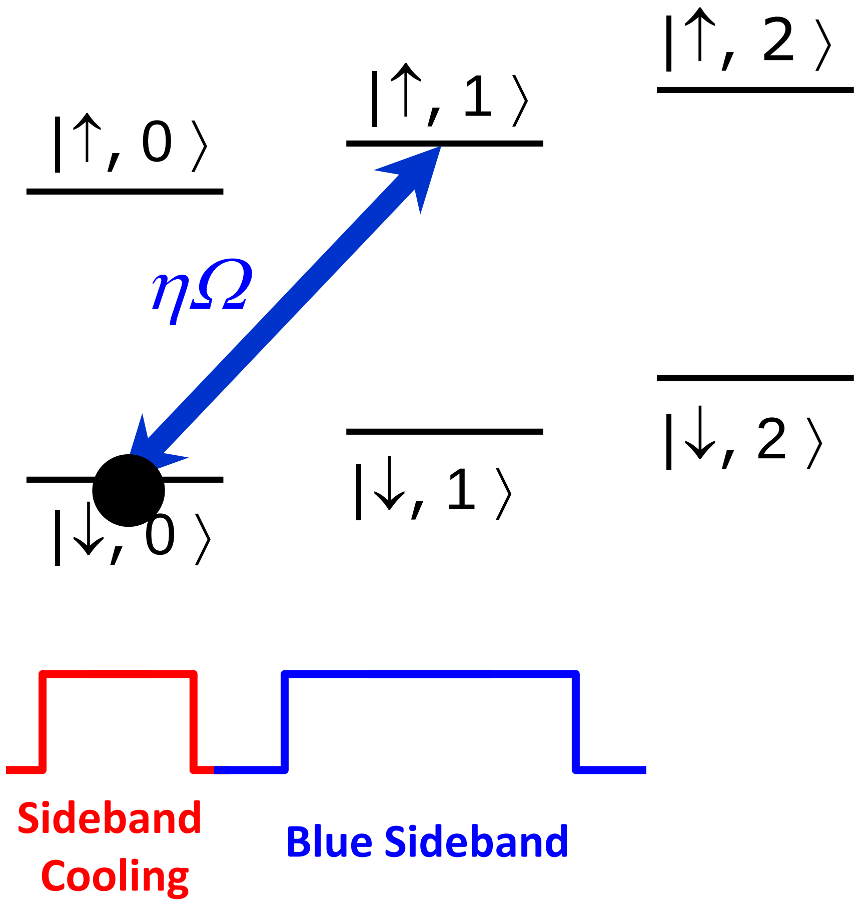
J. Zhang et al., Phys. Rev. A 89, 013608 (2014).
M. V. Berry, J. Phys. A 42, 365303 (2009).

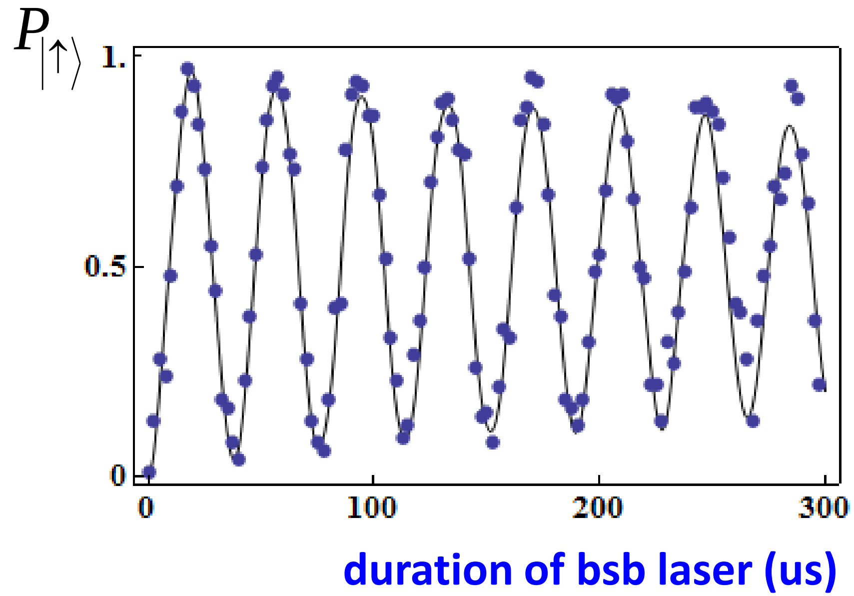
Control of Intensity and Detuning

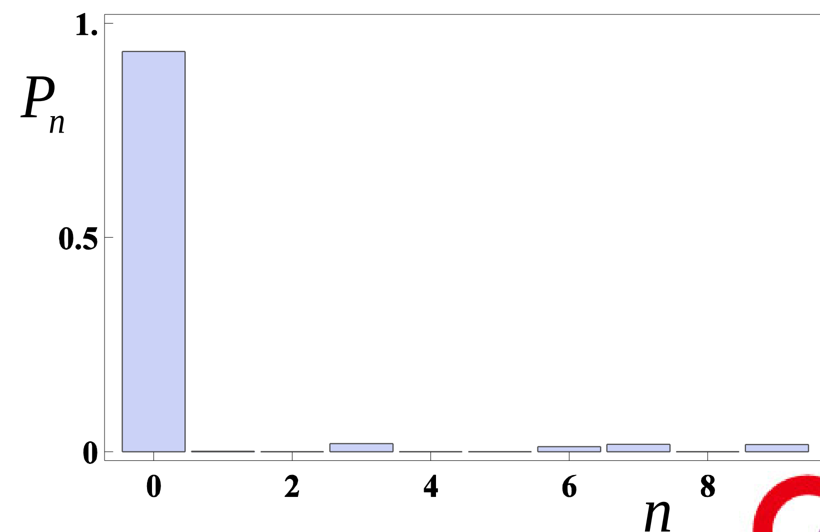


$|n=0\rangle$ Detection

After Sideband Cooling, n=0

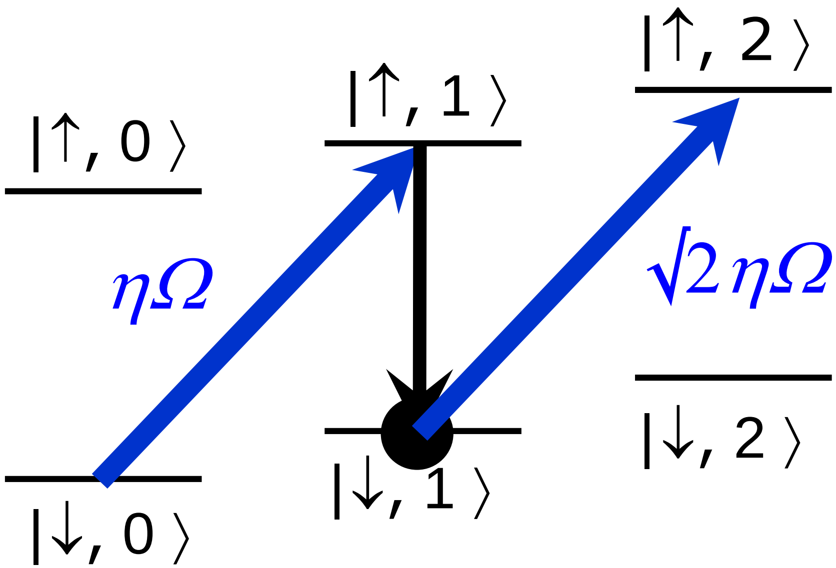


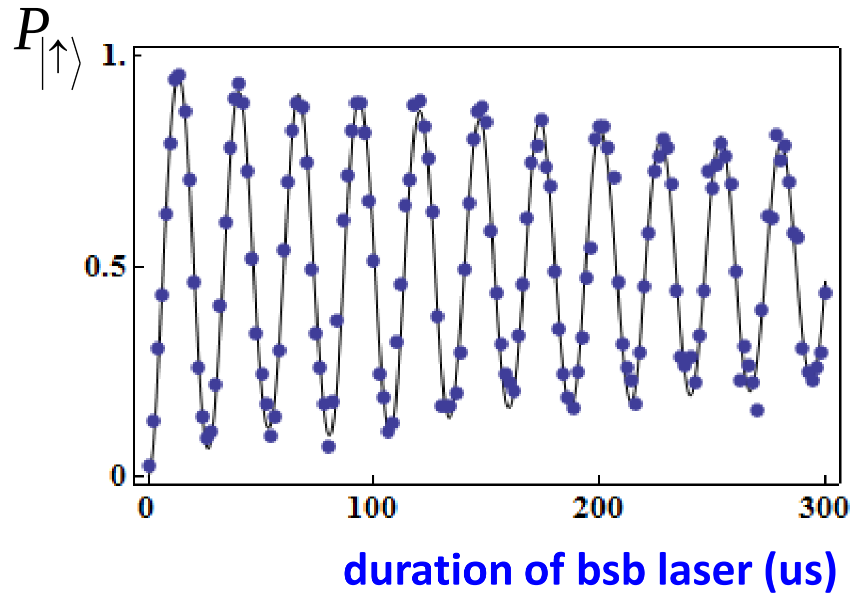


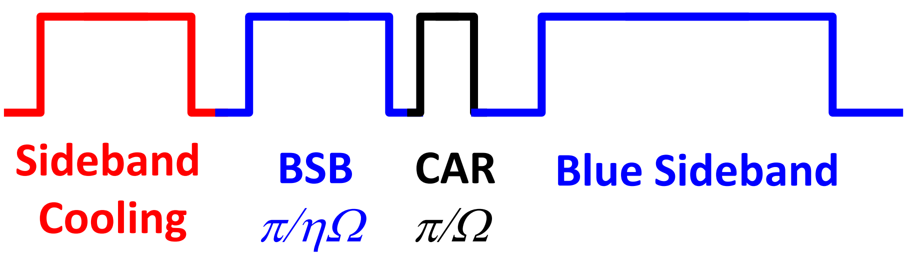


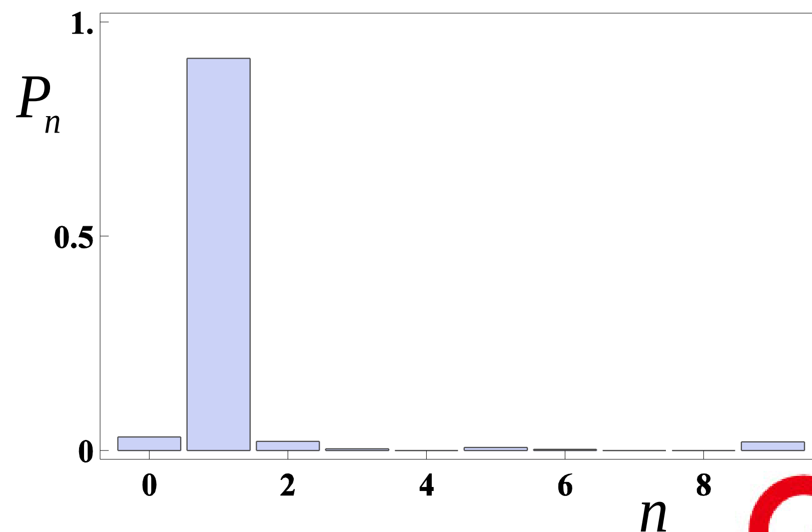
|n=1⟩ State Preparation & Detection

|n=0⟩⟶|n=1⟩

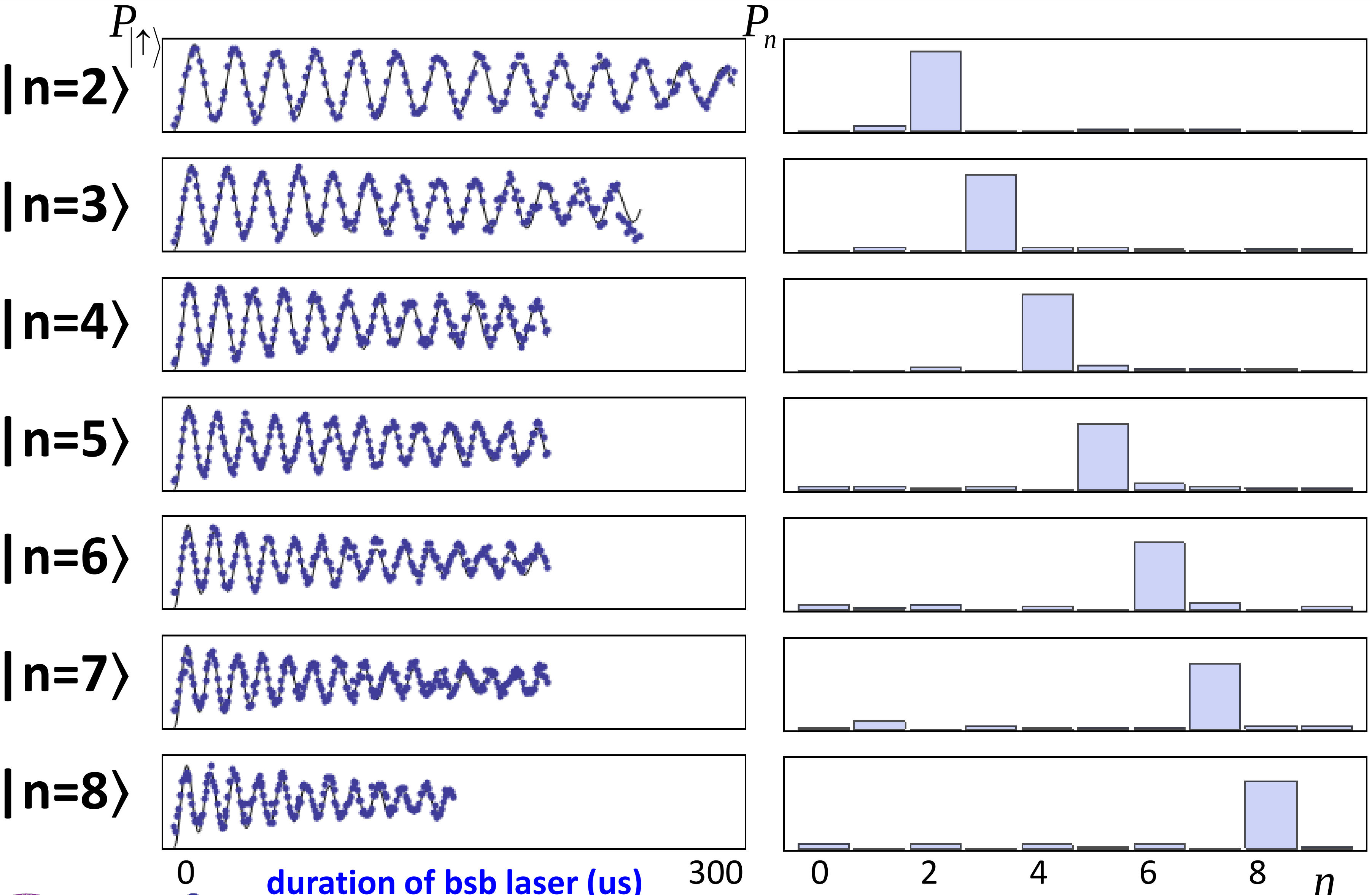




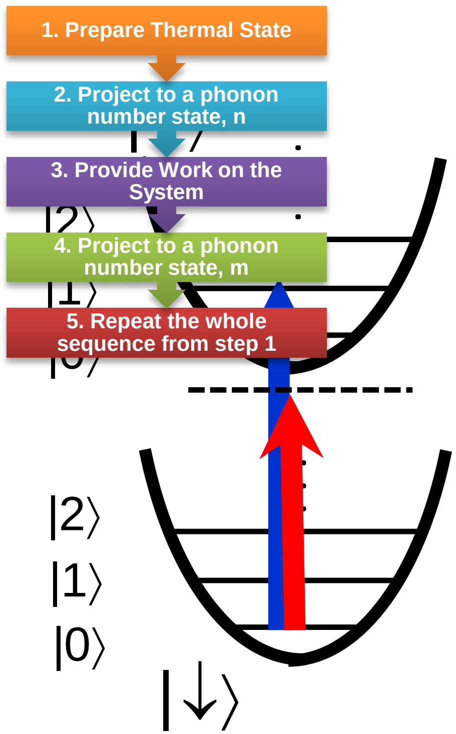




Fock |n⟩ States Preparation



3. Provide Work – Displacement Operation



σ_x Dependent Displacement Operation

P. C. Haljan et al., Phys. Rev. Lett. 94, 153602 (2005).

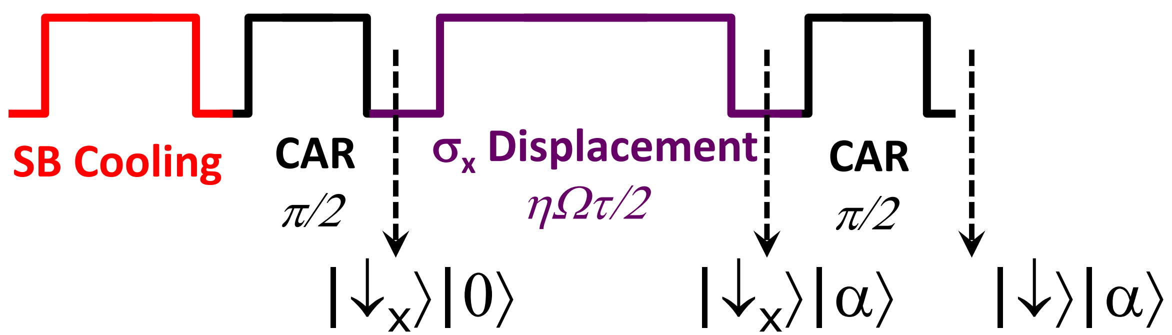
P. J. Lee et al., Journal of Optics B 7, S371 (2005).

$$H_{bsb} = \frac{\eta\Omega}{2}\left(a^{\dagger}\sigma^{+} + a\sigma^{-}\right)$$

$$H_{rsb} = \frac{\eta\Omega}{2}\left(a^{\dagger}\sigma^{-} + a\sigma^{+}\right)$$

$$H_{bsb} + H_{rsb} = \frac{\eta\Omega}{2}\left(a^{\dagger} + a\right)\sigma_{x}$$

Pure Displacement Operation

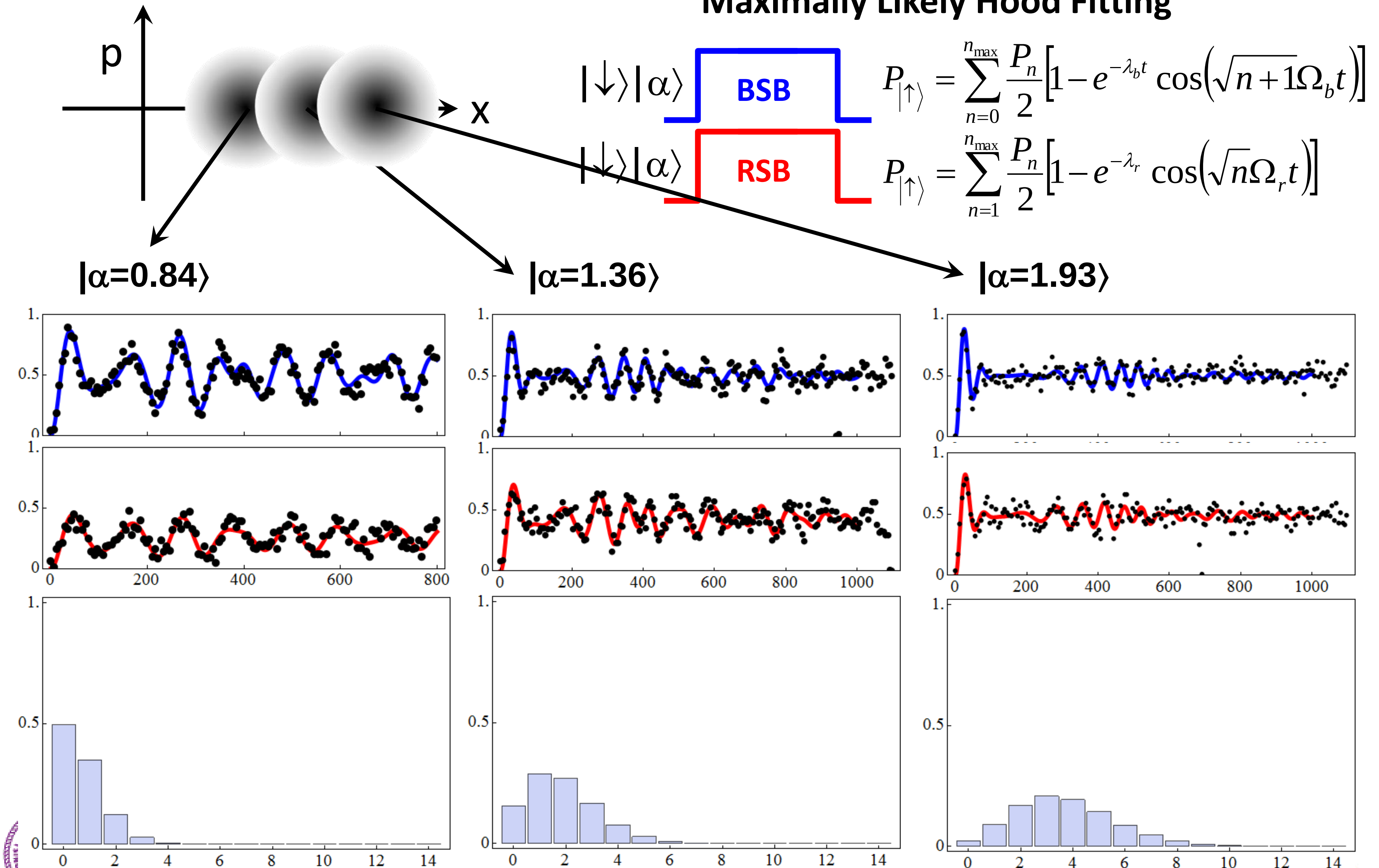


Coherent State Detection

Maximally Likely Hood Fitting

$$|\downarrow\rangle|\alpha\rangle \quad \text{BSB} \qquad P_{|\uparrow\rangle} = \sum_{n=0}^{n_{\max}} \frac{P_n}{2}\left[1 - e^{-\lambda_b t}\cos\left(\sqrt{n+1}\,\Omega_b t\right)\right]$$

$$|\downarrow\rangle|\alpha\rangle \quad \text{RSB} \qquad P_{|\uparrow\rangle} = \sum_{n=1}^{n_{\max}} \frac{P_n}{2}\left[1 - e^{-\lambda_r}\cos\left(\sqrt{n}\,\Omega_r t\right)\right]$$

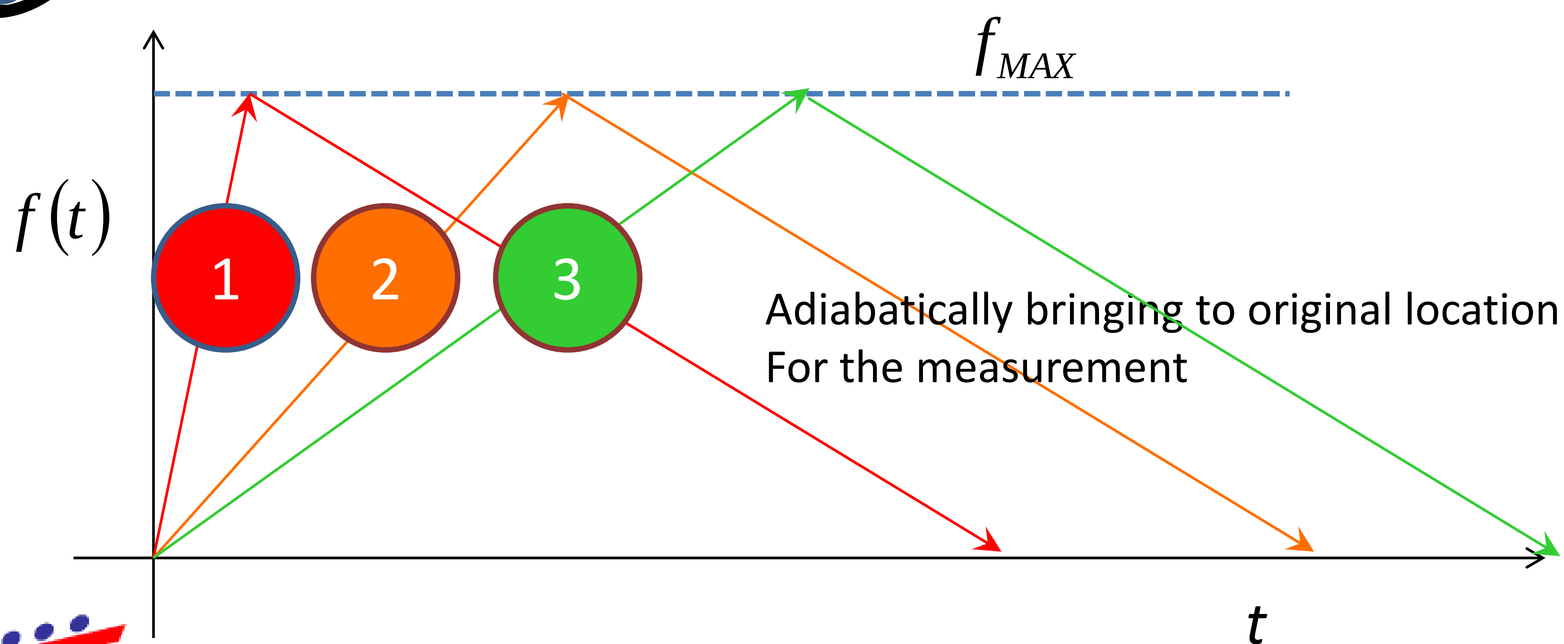


Work in Our System

In the rotating frame with respect to the driving laser frequency

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\delta^2 \hat{x}^2 + f(t)\hat{x} = \hbar\delta\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \frac{x_0 f(t)}{\sqrt{2}}\left(\hat{a}^\dagger + \hat{a}\right),$$

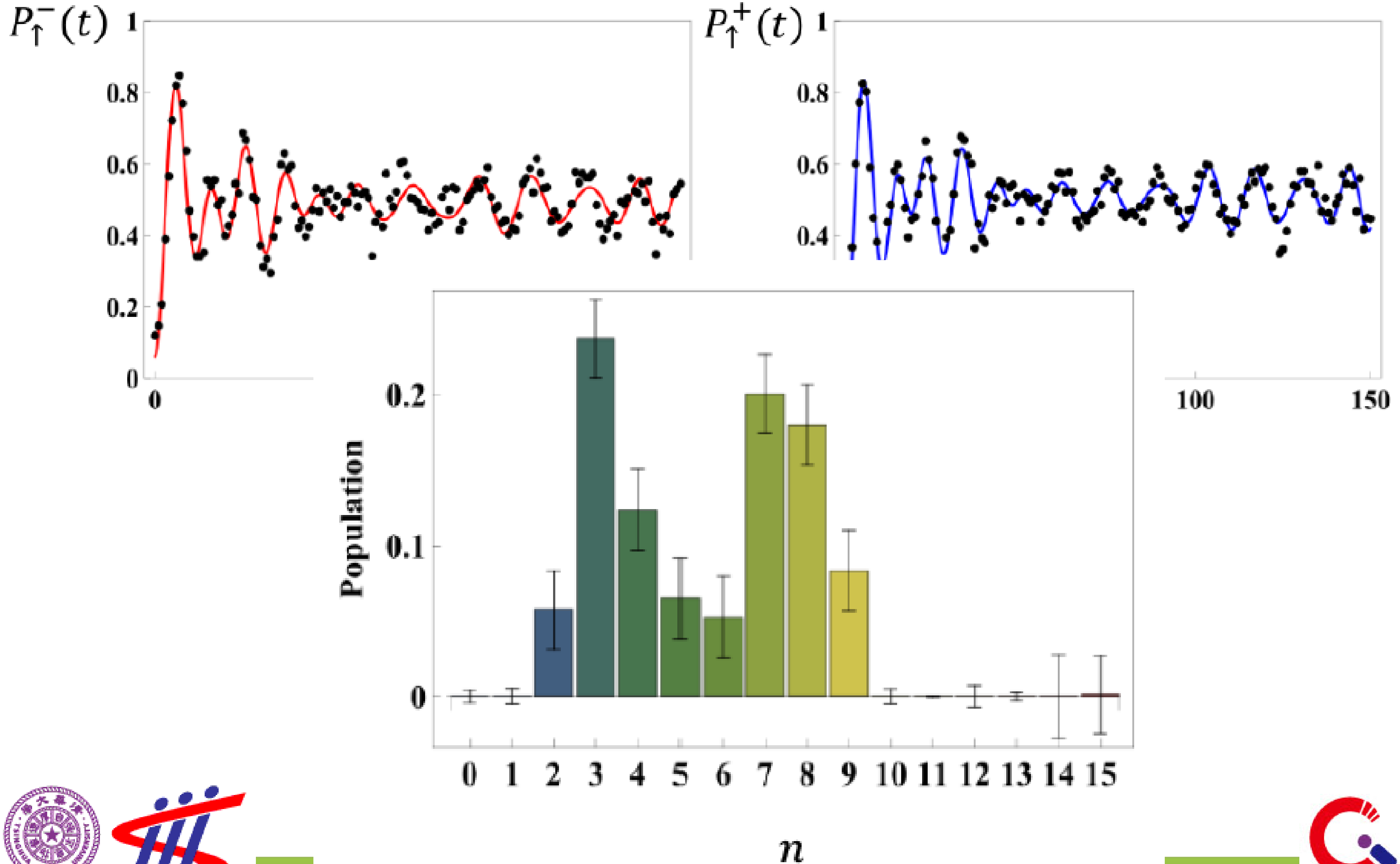
$$\text{where } \delta = \omega_t - \omega_L,\ x_0 = \sqrt{\frac{\hbar}{m\omega_t}}$$



Final State Measurements – Fitting Methods

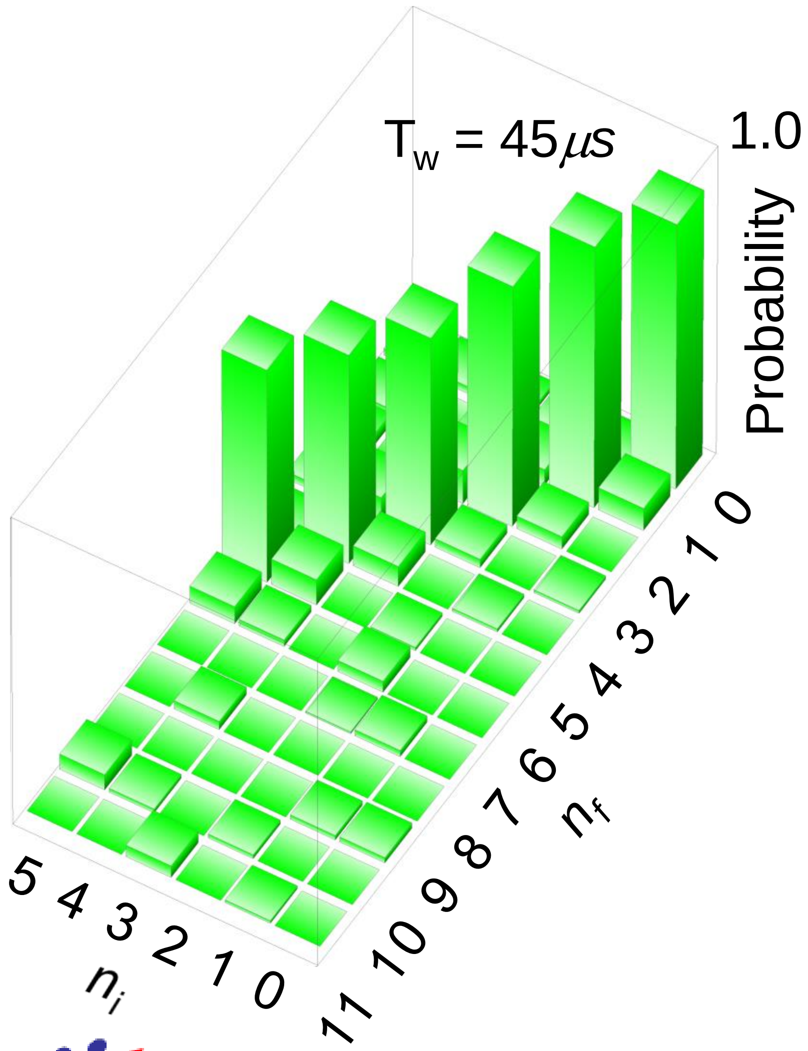
1. Prepare Thermal State

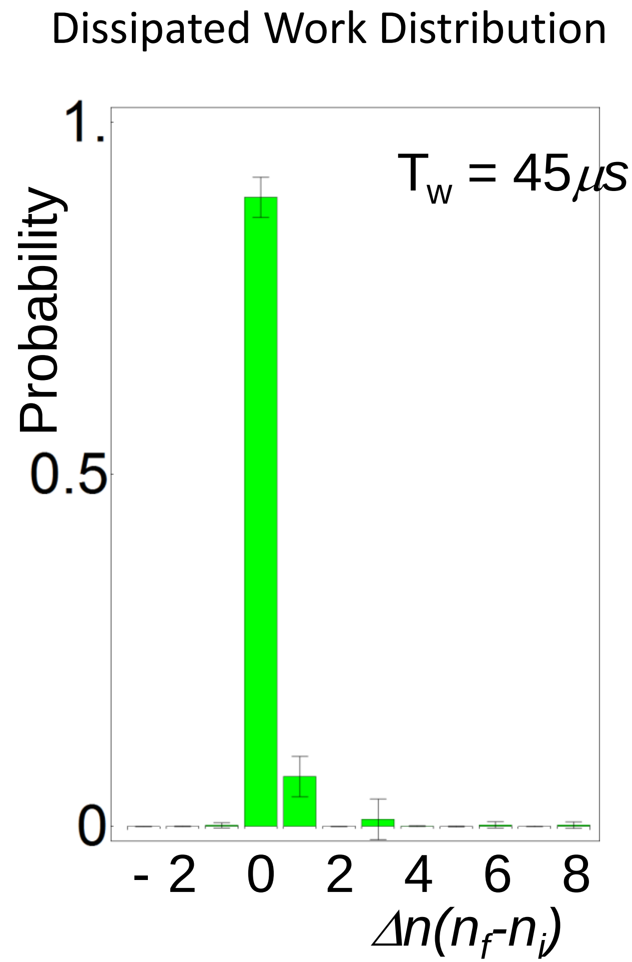
eg.) n_i=5, 25 μs



Final State Measurements – Intermediate Work

3

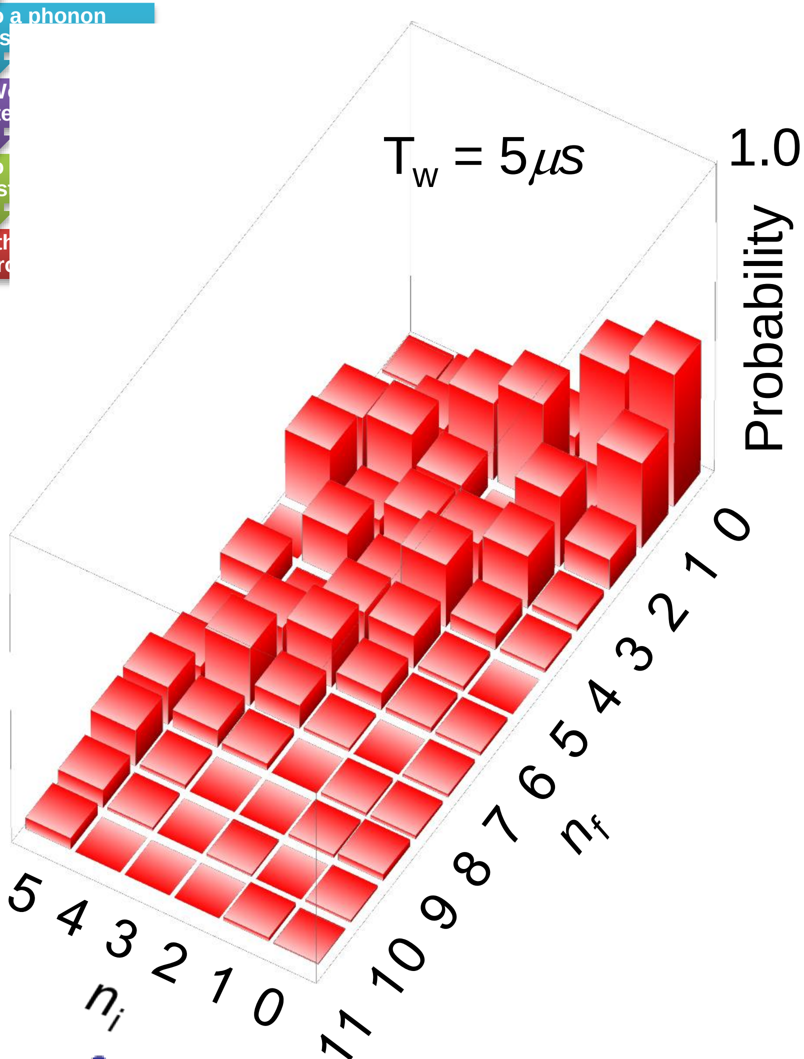


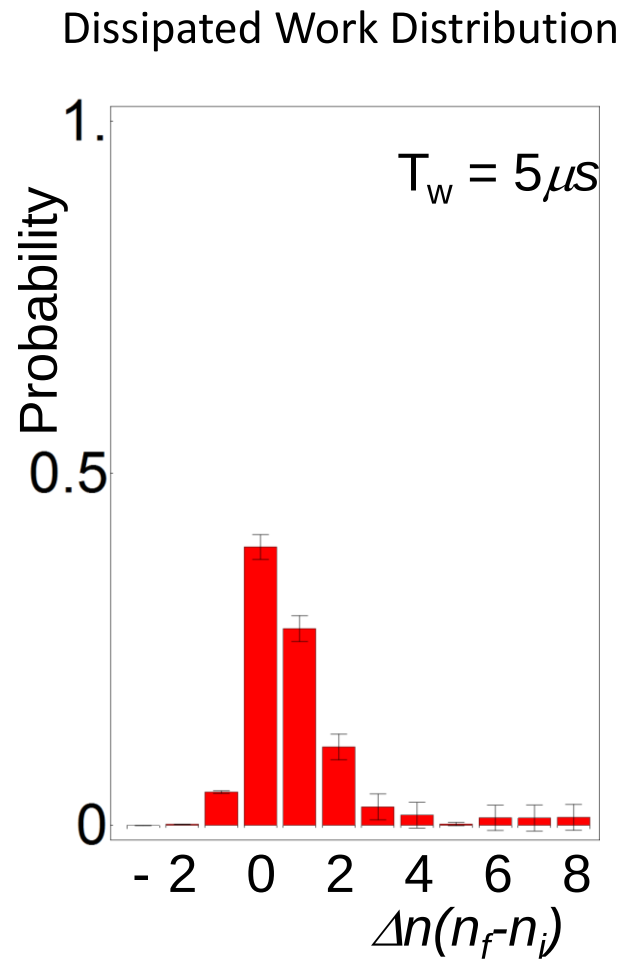


$$\left\langle e^{-\beta W-\beta\Delta F}\right\rangle = 0.989 \pm 0.072$$

Final State Measurements – Non equilibrium Work

1. Prepare Thermal State
2. Project to a phonon number s
3. Provide W Syste
4. Project to number s
5. Repeat th sequence fr

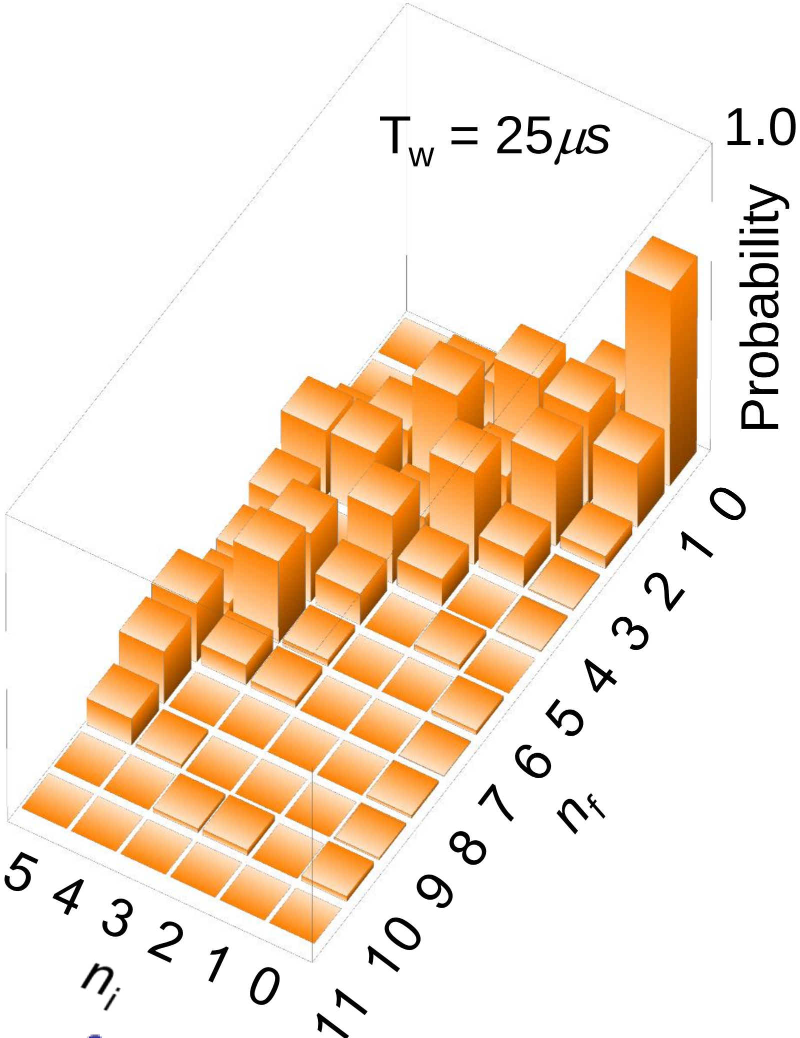




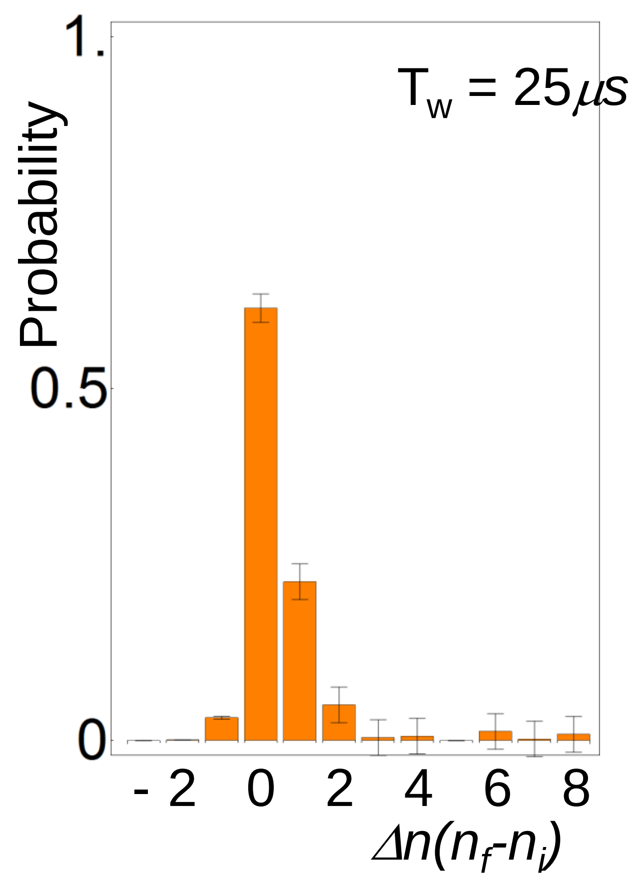
$$\left\langle e^{-\beta W-\beta\Delta F}\right\rangle = 1.032 \pm 0.038$$

Final State Measurements – Intermediate Work

2



Dissipated Work Distribution



$$\left\langle e^{-\beta W-\beta\Delta F}\right\rangle = 0.995 \pm 0.045$$

Comparison to other estimations

$\Delta F/k_B T_{\mathrm{eff}}$	$-\ln\left\langle e^{-W_{\mathrm{diss}}/k_B T_{\mathrm{eff}}}\right\rangle$		
	τ =5 μs	τ =25 μs	τ =45 μs
-2.63 (316 nK)	-0.032(±37)	0.006(±34)	0.042(±52)
-2.13 (390 nK)	-0.033(±35)	0.005(±33)	0.037(±50)
-1.73 (480 nK)	-0.034(±34)	0.003(±31)	0.031(±48)

$\Delta F/k_B T_{\mathrm{eff}}$	$\langle W_{\mathrm{diss}}/k_B T_{\mathrm{eff}}\rangle - \frac{1}{2}\frac{\sigma^2}{(k_B T_{\mathrm{eff}})^2}$		
	τ =5 μs	τ =25 μs	τ =45 μs
-2.63 (316 nK)	-1.601(±443)	-0.718(±568)	-0.087(±154)
-2.13 (390 nK)	-0.889(±346)	-0.426(±442)	-0.027(±120)
-1.73 (480 nK)	-0.505(±269)	-0.260(±342)	0.002(±93)

$\Delta F/k_B T_{\mathrm{eff}}$	$\langle W_{\mathrm{diss}}/k_B T_{\mathrm{eff}}\rangle$		
	τ =5 μs	τ =25 μs	τ =45 μs
-2.63 (316 nK)	2.573(±313)	0.929(±401)	0.211(±109)
-2.13 (390 nK)	2.033(±245)	0.749(±313)	0.168(±85)
-1.73 (480 nK)	1.598(±190)	0.602(±242)	0.131(±66)

Conclusion and Outlook

- We experimentally verify the Quantum Jarzynski Equality with our Trapped ion system

- It could be extended to the verification of the Equality in open quantum system

Thank you for your attention!