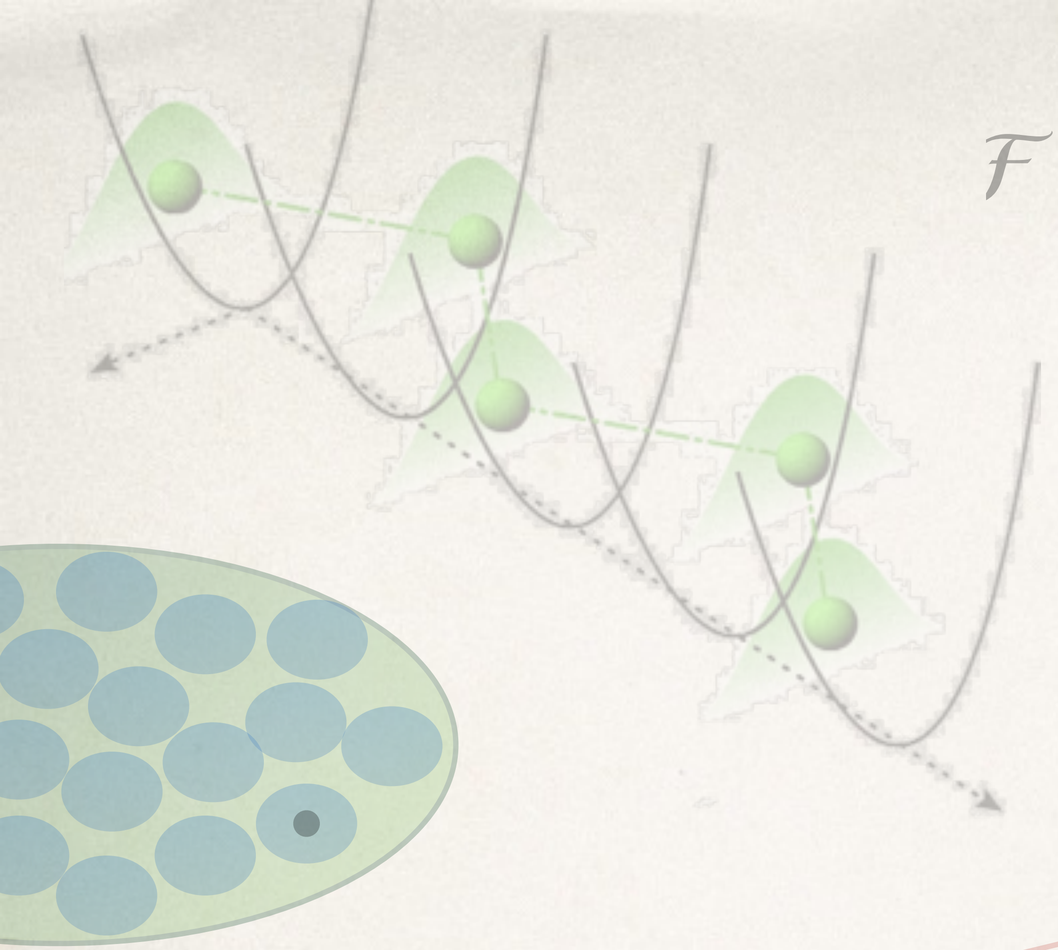
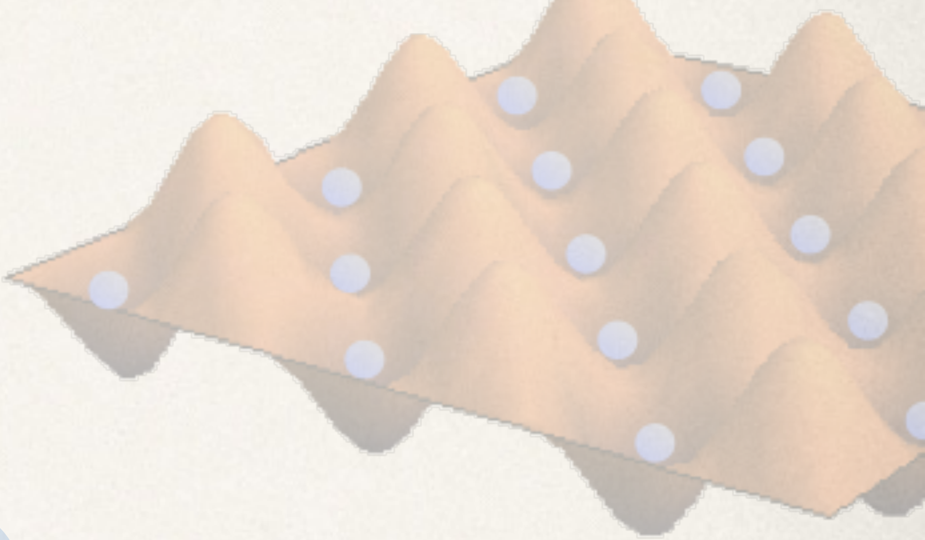
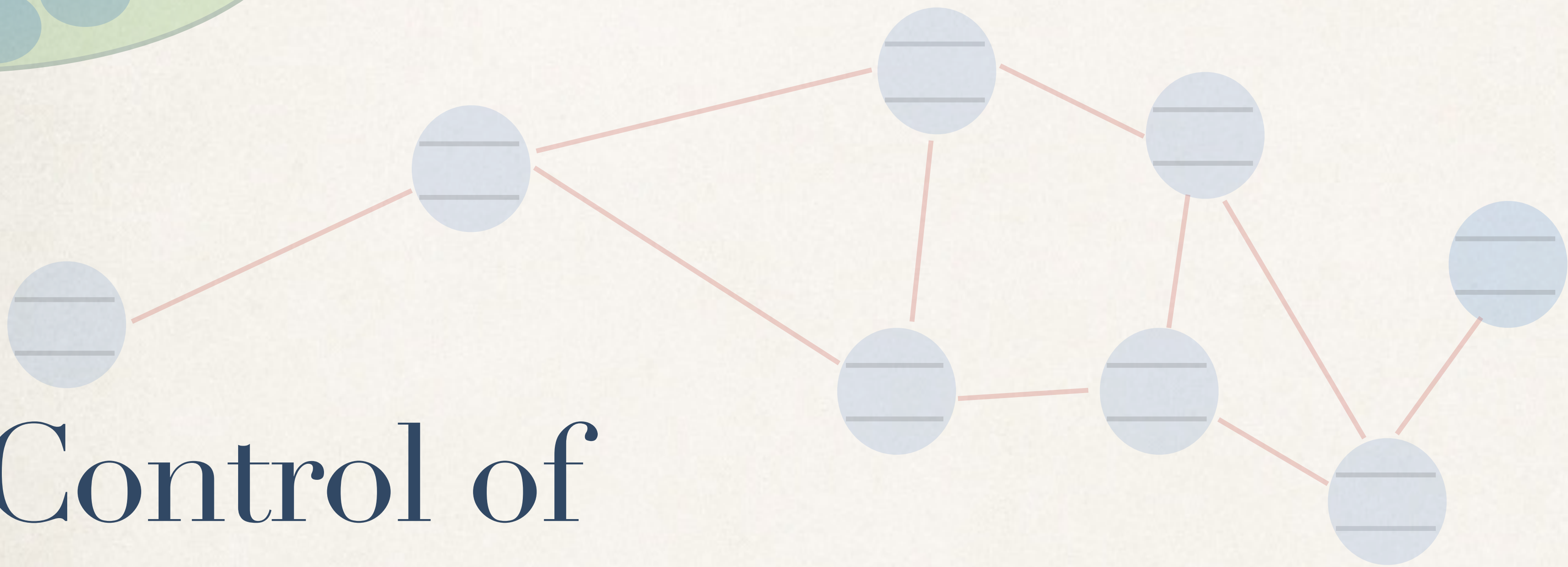
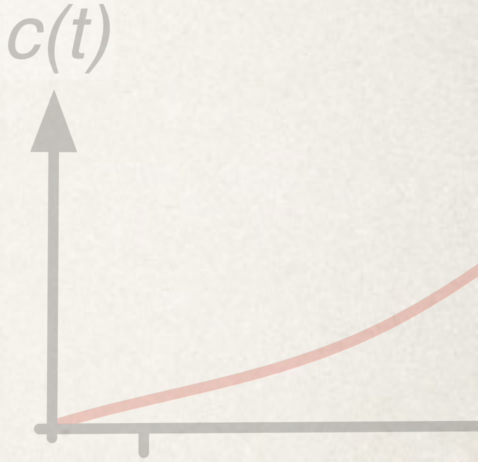




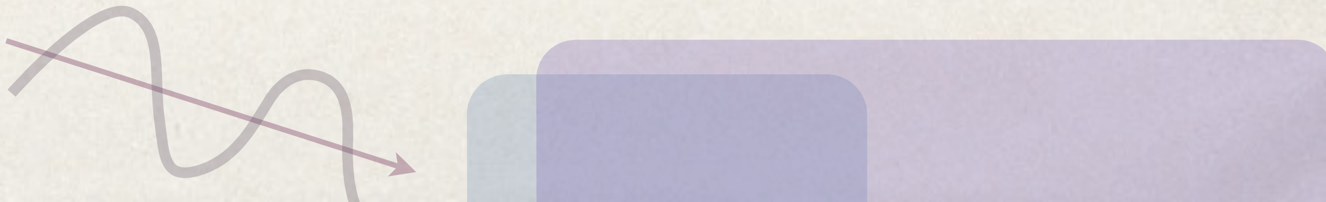




Control of many-body quantum systems

Simone Montangero - Ulm University

Telluride - 15/7/2014



Simulation and control

- Quantum Information Processing (computation, communication etc.)
- Quantum simulators benchmarking and validation
- Design and validation of experiments with quantum hardware
- HEP investigations?

Hardware description

SIMULATION

Software development

CONTROL

Many-body quantum systems

Time-dependent Schrödinger equation

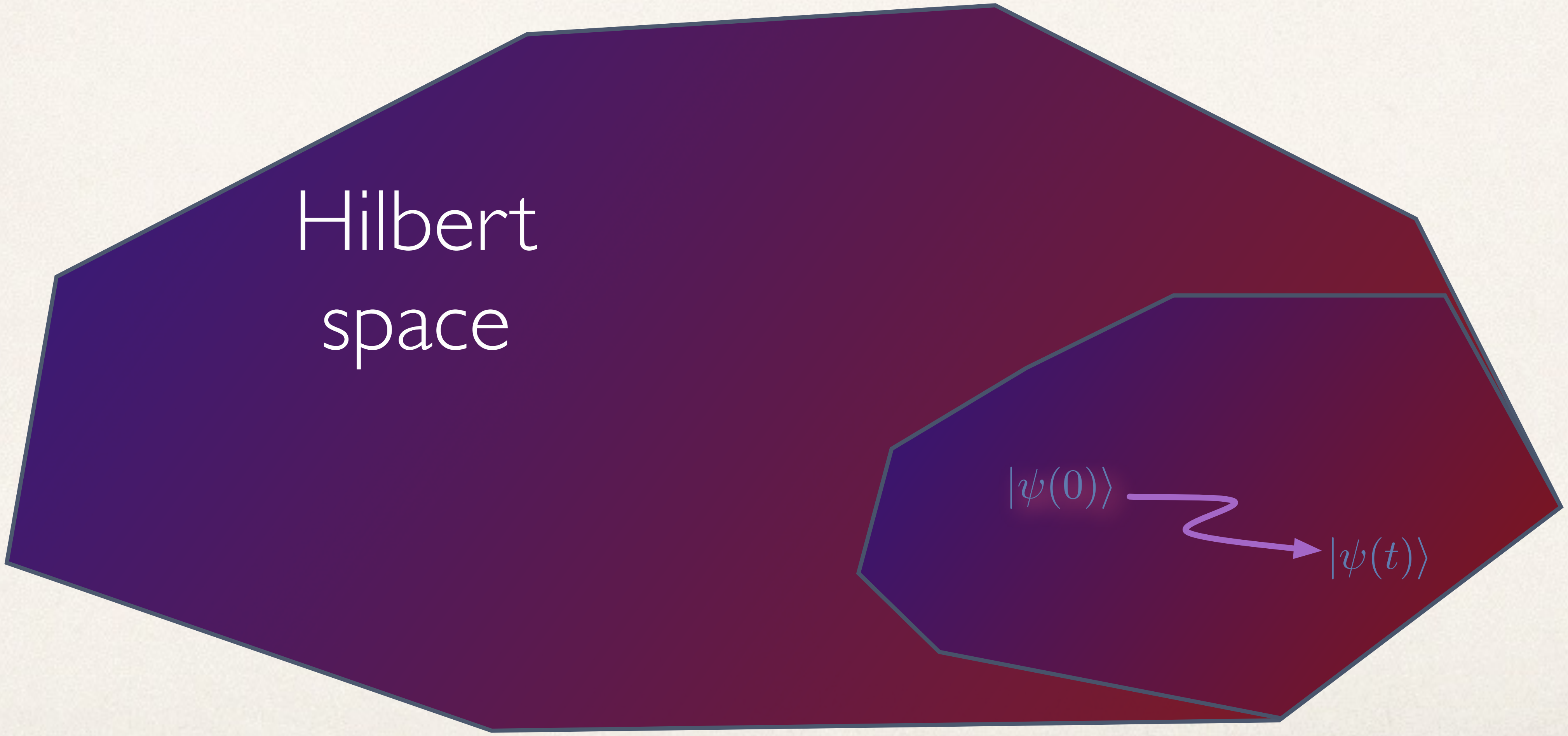
$\mathcal{H}_{loc} \otimes \mathcal{H}_{loc} \otimes \mathcal{H}_{loc}$

$+H_{int}$ $+H_{int}$

2^N

Tensor Network methods

"Simple" Ansatz to describe faithfully "interesting" quantum states!

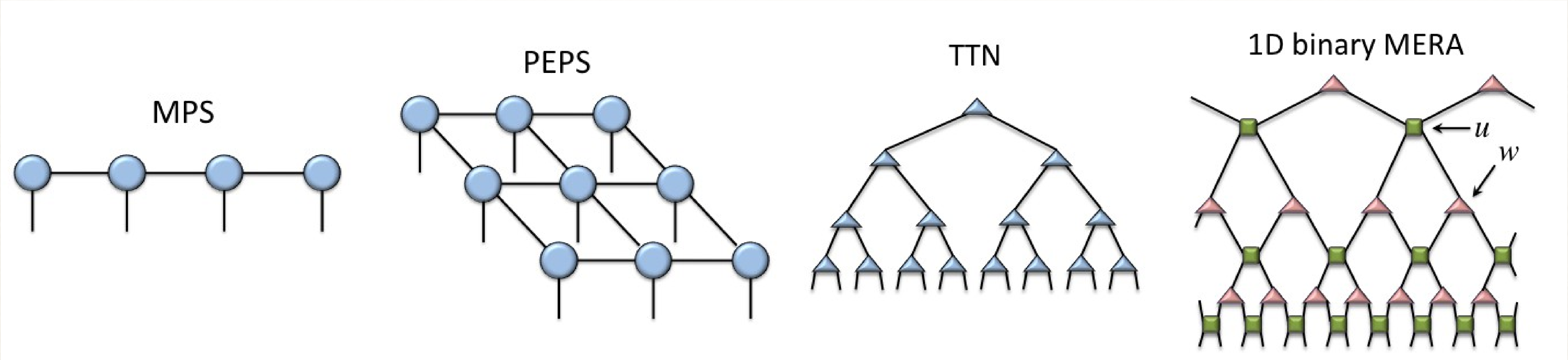


Tensor Network Methods

$$|\psi\rangle = \sum_{\vec{\alpha}} \psi_{\alpha_1,\alpha_2,\ldots\alpha_n} |\alpha_1\rangle|\alpha_2\rangle \ldots |\alpha_n\rangle$$

$|\alpha_\imath\rangle$

local basis



$$|\psi\rangle = \sum_{\vec{\alpha}} A^{\beta_1}_{\alpha_1} A^{\beta_1\beta_2}_{\alpha_2} \ldots A^{\beta_{n-1}}_{\alpha_n} |\alpha_1\rangle|\alpha_2\rangle \ldots |\alpha_n\rangle$$

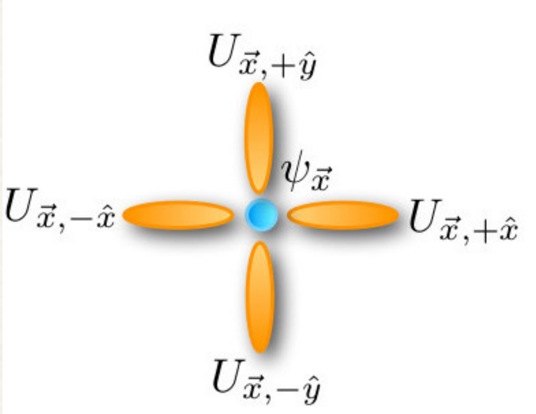
Polynomial efforts in 1D

U. Schollwock, Rev. Mod. Phys. (2005)

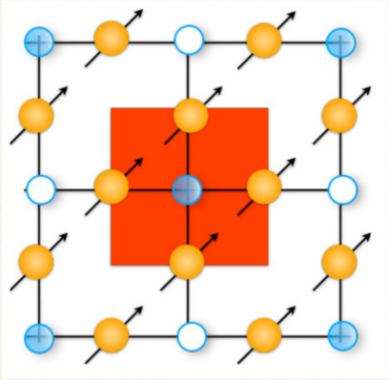
Lattice Gauge Theories

- Site d.o.f. (matter fields..)
- Quantum link d.o.f. (bosons, spins,..)
- Gauge invariant Hamiltonian

Strongly correlated
models at low energies
(spin ice, quantum dimer..)



Non-perturbative
HEP models
(QED, QCD)



Atomic quantum
simulators

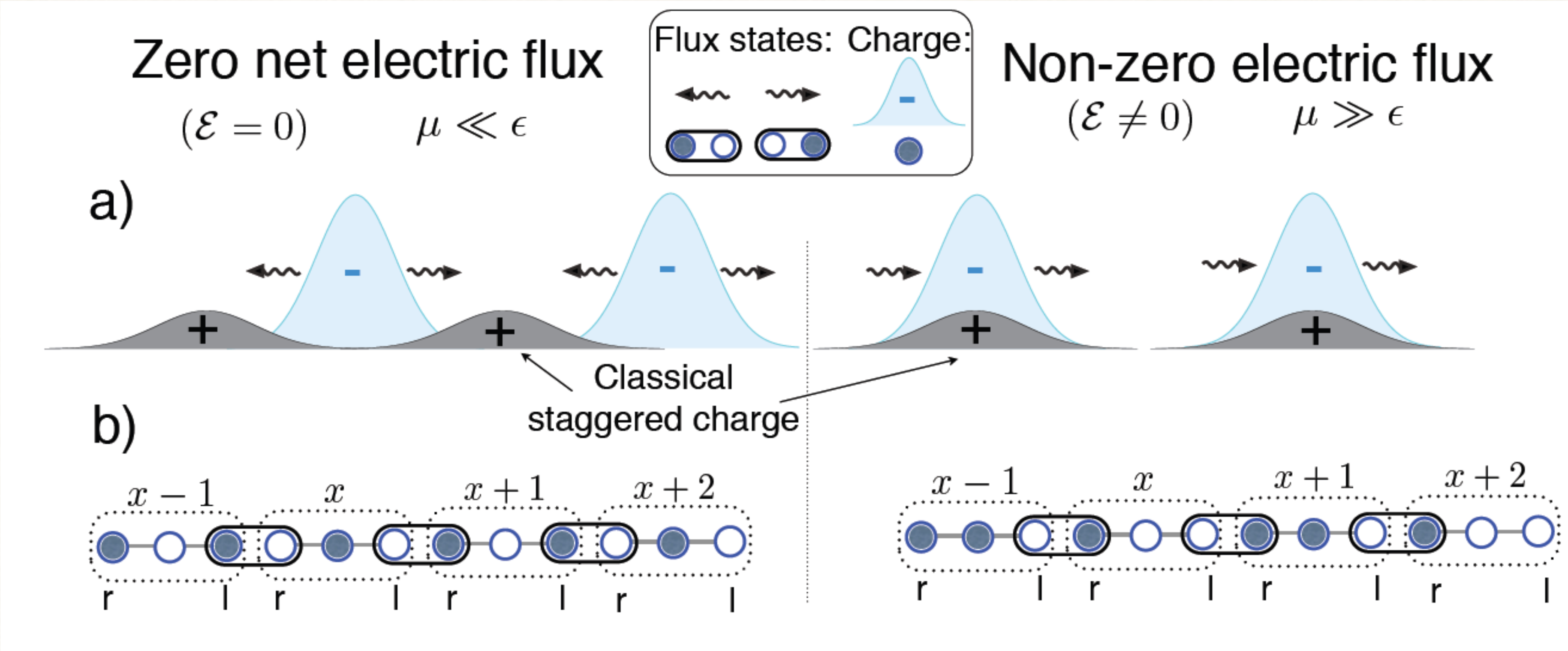
Lattice gauge tensor network
simulations
(abelian and non abelian)

E.Rico, T. Pichler, M. Dalmonte,
P. Zoller, SM, PRL (2014)

P. Silvi, E.Rico, T. Calarco, SM, arXiv:1404.7439

Schwinger model in 1+1D

$$H = \sum_x (E_{x,x+1} - (-1)^x E_0)^2 + \mu(-1)^x \psi_x^\dagger \psi_x - \epsilon(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{h.c.})$$



Disordered phase

Quantum phase transition

Ordered phase symmetry breaking

E.Rico, T. Pichler, M. Dalmonte, P. Zoller, SM, PRL (2014)

Optimal control

- Few-body quantum systems: standard optimal control (high-accuracy, complete knowledge, many iterations...)

- Many-body?

H. Rabitz, NJP (2009)
Altafini & Ticozzi IEEE (2012)

Chopped RAndom Basis

Expand control field over n_f
"randomized" basis functions

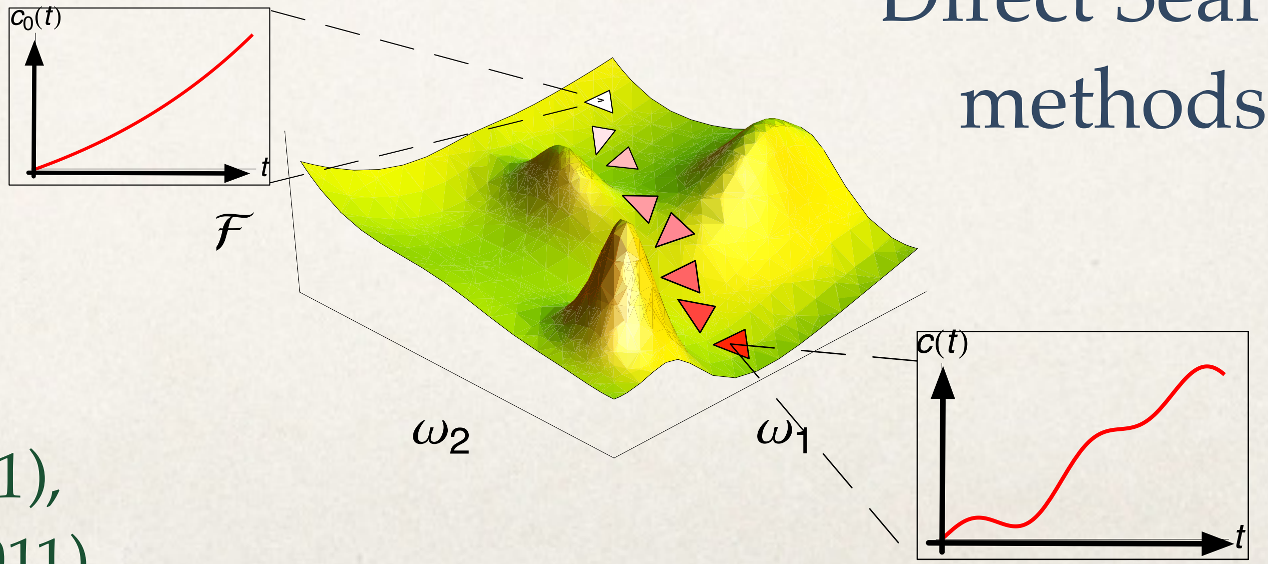
Reduced basis method

Multivariable
function minimization

Functional
minimization

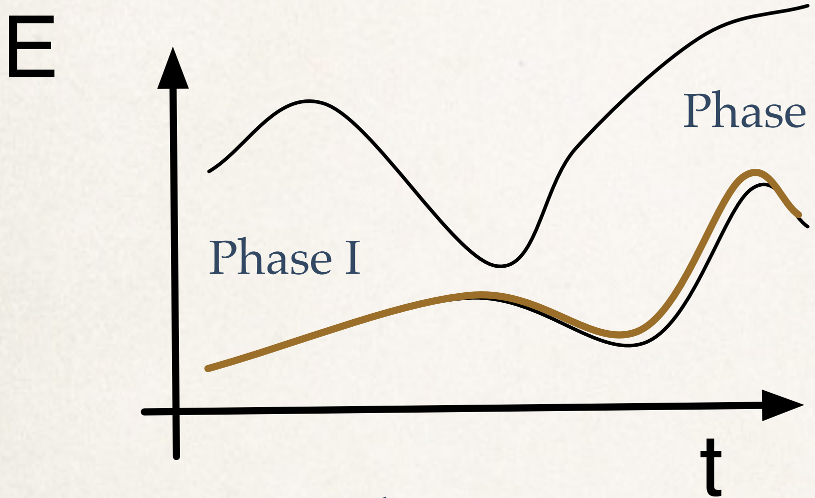
Direct Search
methods

O(10) parameters!

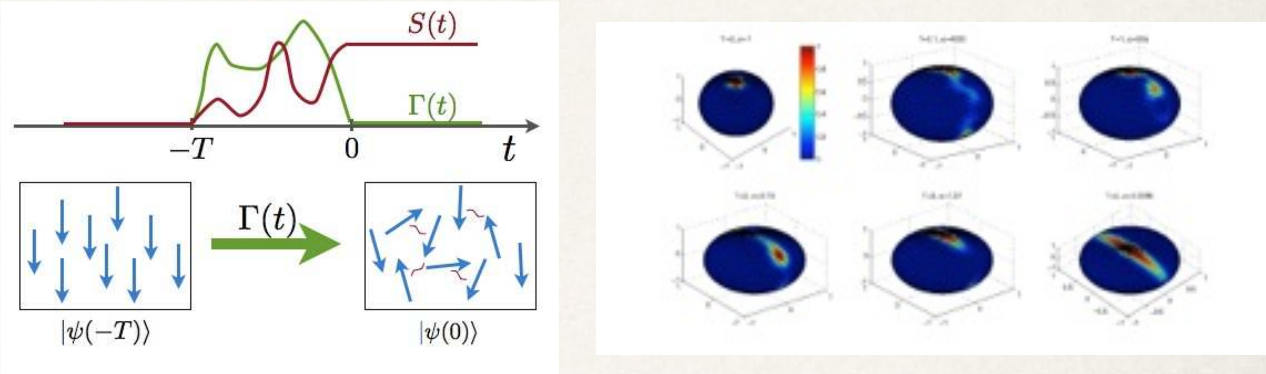


P. Doria, T. Calarco, SM PRL (2011),
T. Caneva, T. Calarco, SM, PRA (2011)

Applications

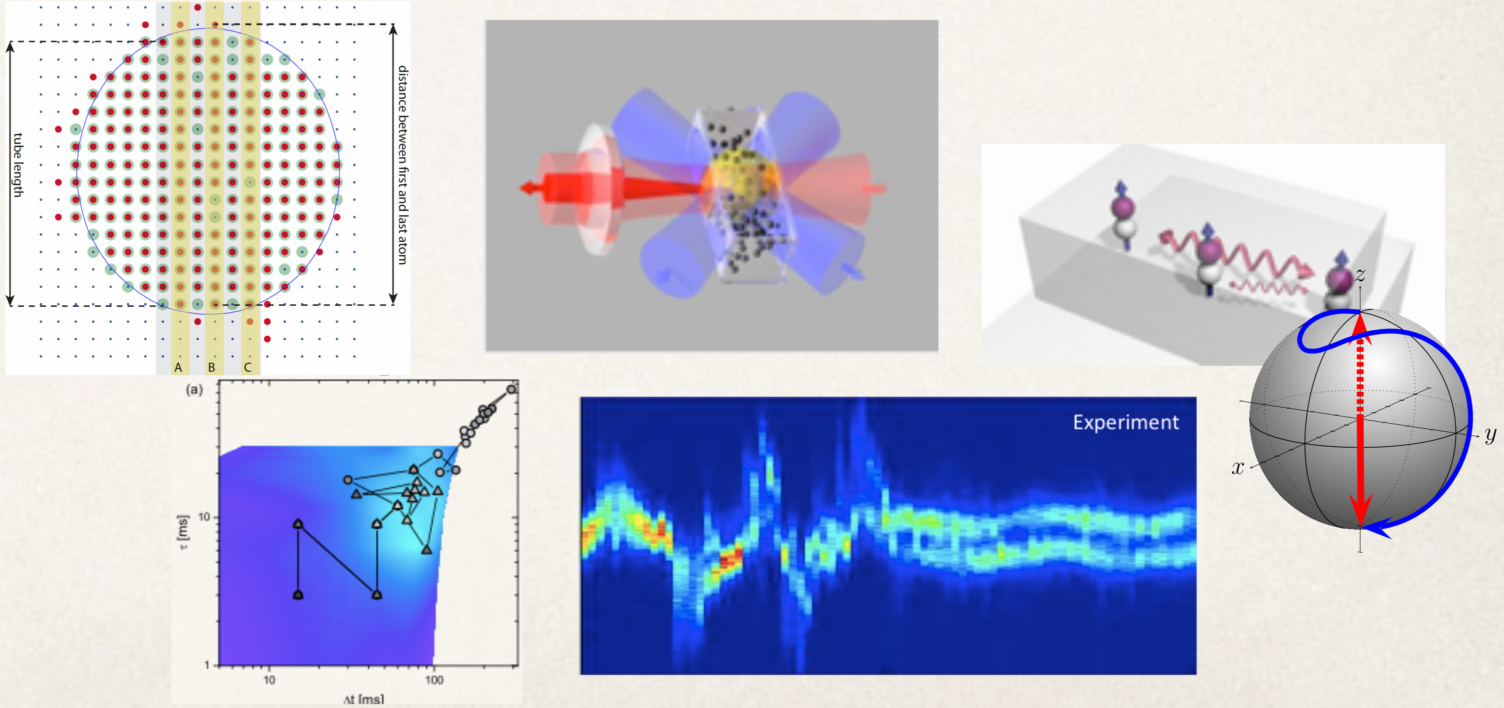


Quantum Phase Transition dynamics



Entanglement/Squeezing manipulation

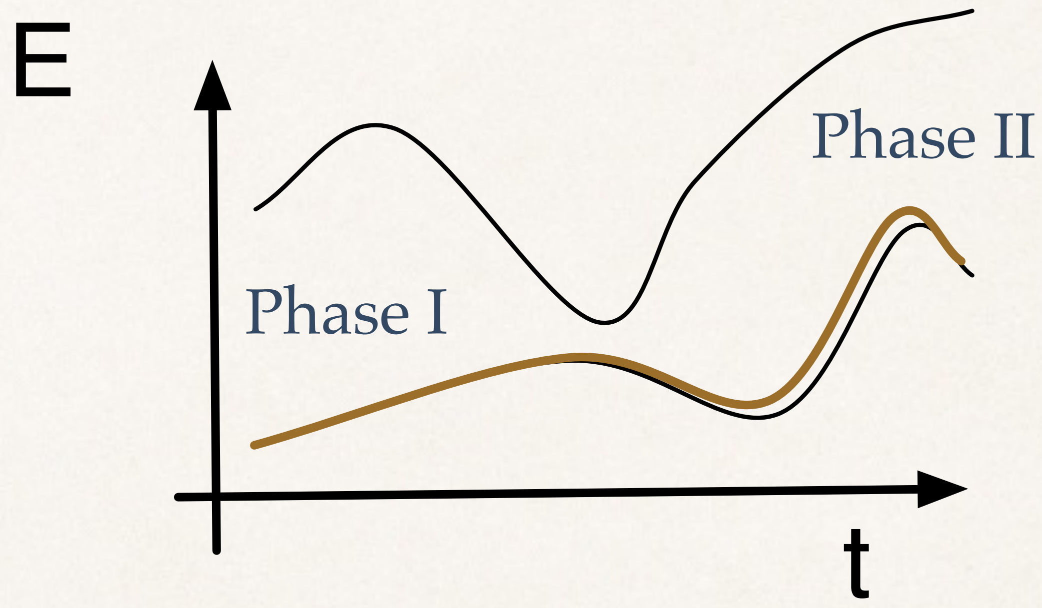
Light-harvesting dynamics



Optimal experimental protocols

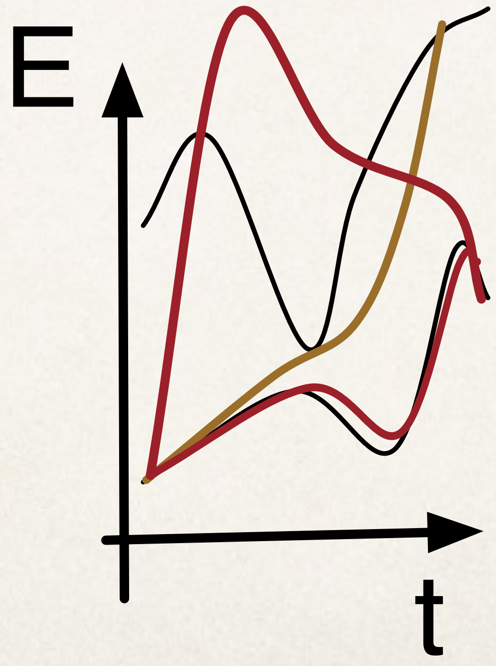
Control of QPT dynamics

Slow



Adiabatic strategy

Fast

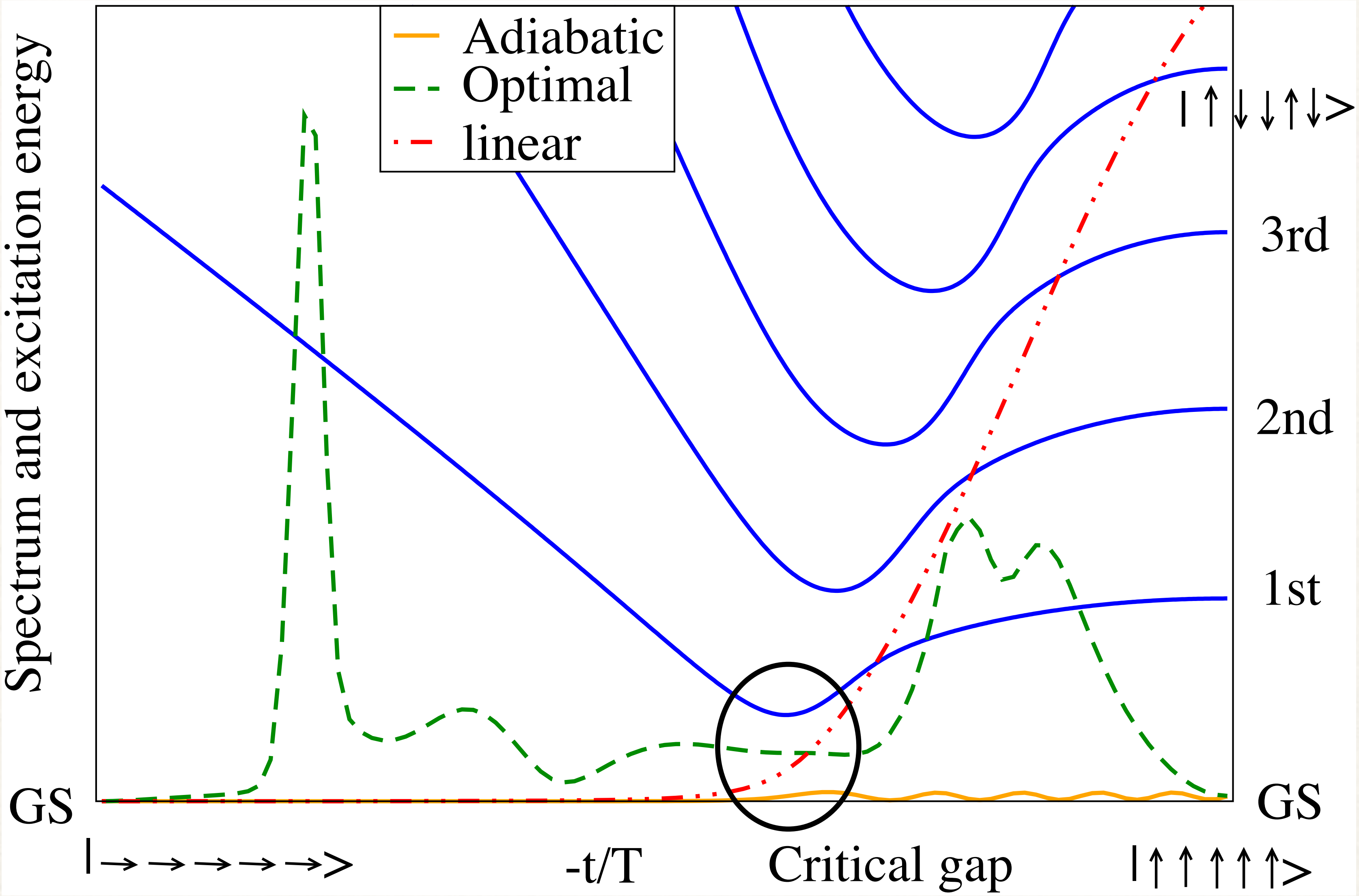


Shortcuts

Optimal control

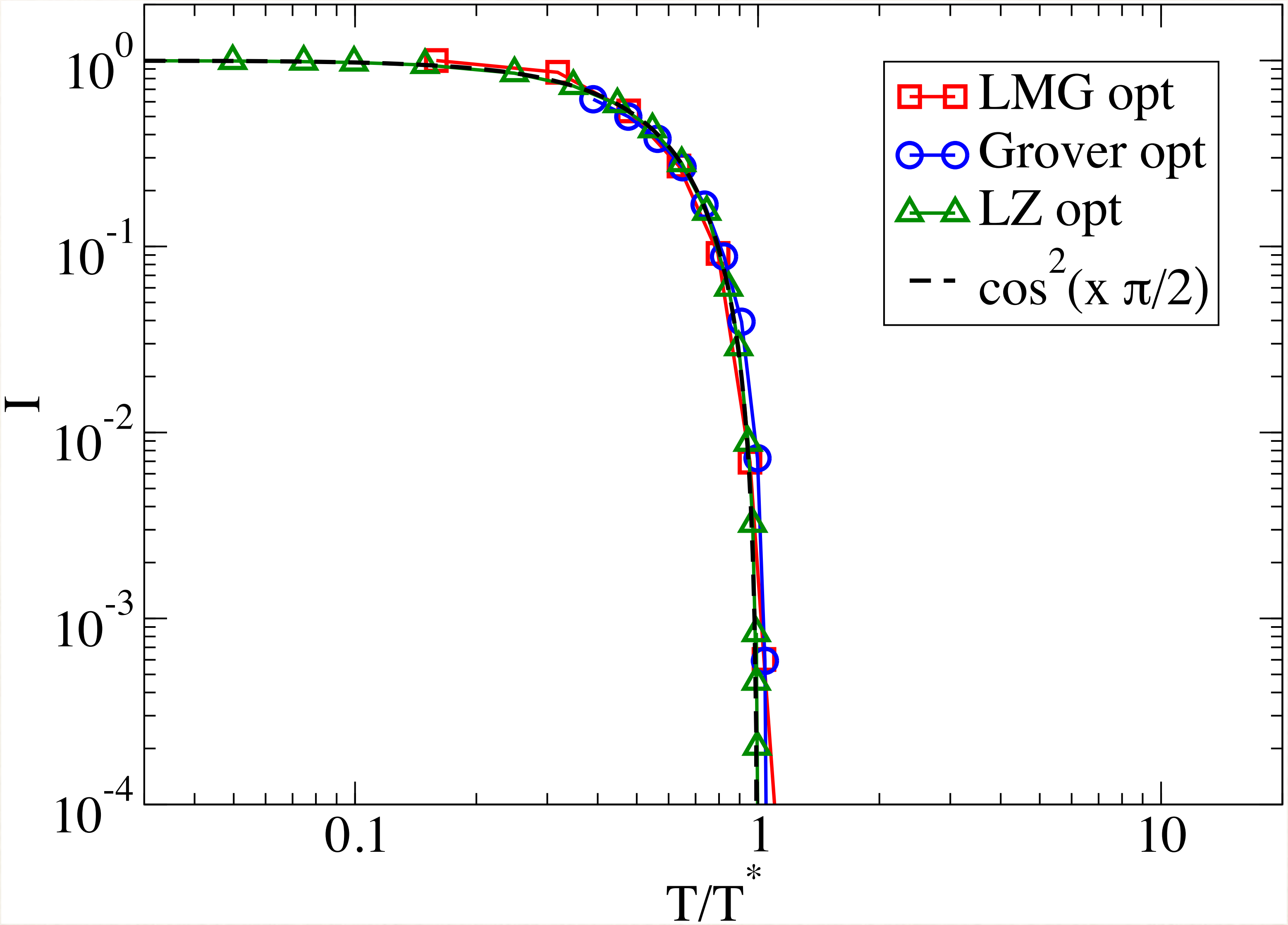
Optimal QPT crossing

LMG model



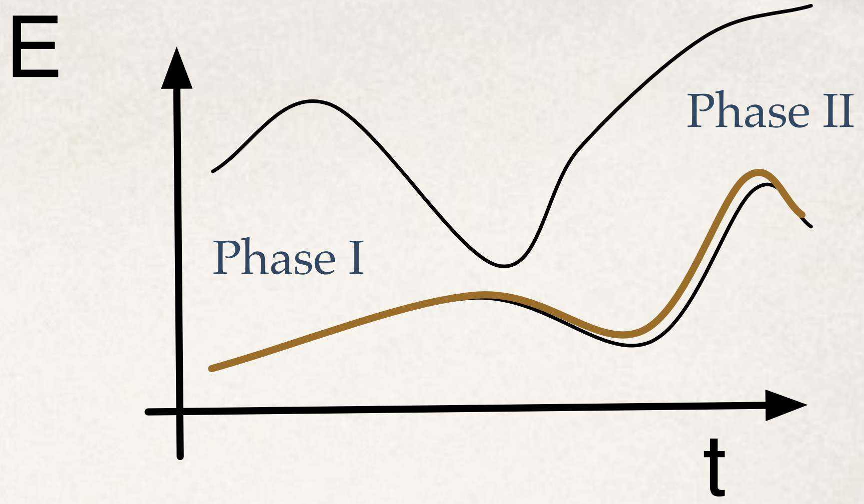
$$H = -\frac{1}{N}\sum_{i<j}(\sigma_i^x\sigma_j^x + \gamma\sigma_i^y\sigma_j^y) - \Gamma(t)\sum_i^N \sigma_i^z$$

Quantum speed limit



see also T. Caneva, M. Murphy, T. Calarco, R. Fazio, SM, V. Giovannetti, and G. E. Santoro, Phys. Rev. Lett. 103, 240501 (2009).

QPT crossing scenario



$T \ll \Delta^{-1}$ $T \sim \Delta^{-1}$ $T \gg \Delta^{-1}$

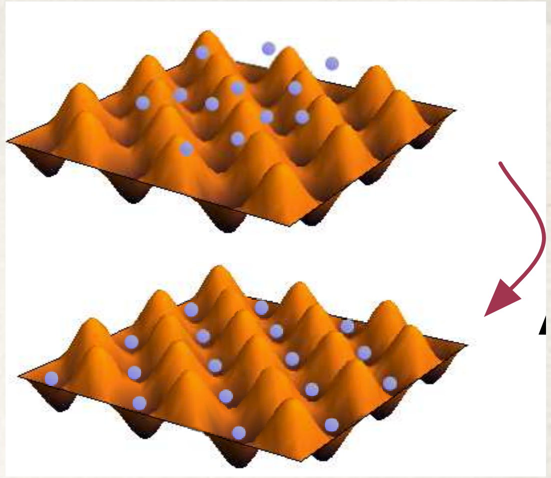
Linear s: $I \sim O(1)$ | π | $I(s \gg 1) \to 0$ | ∞

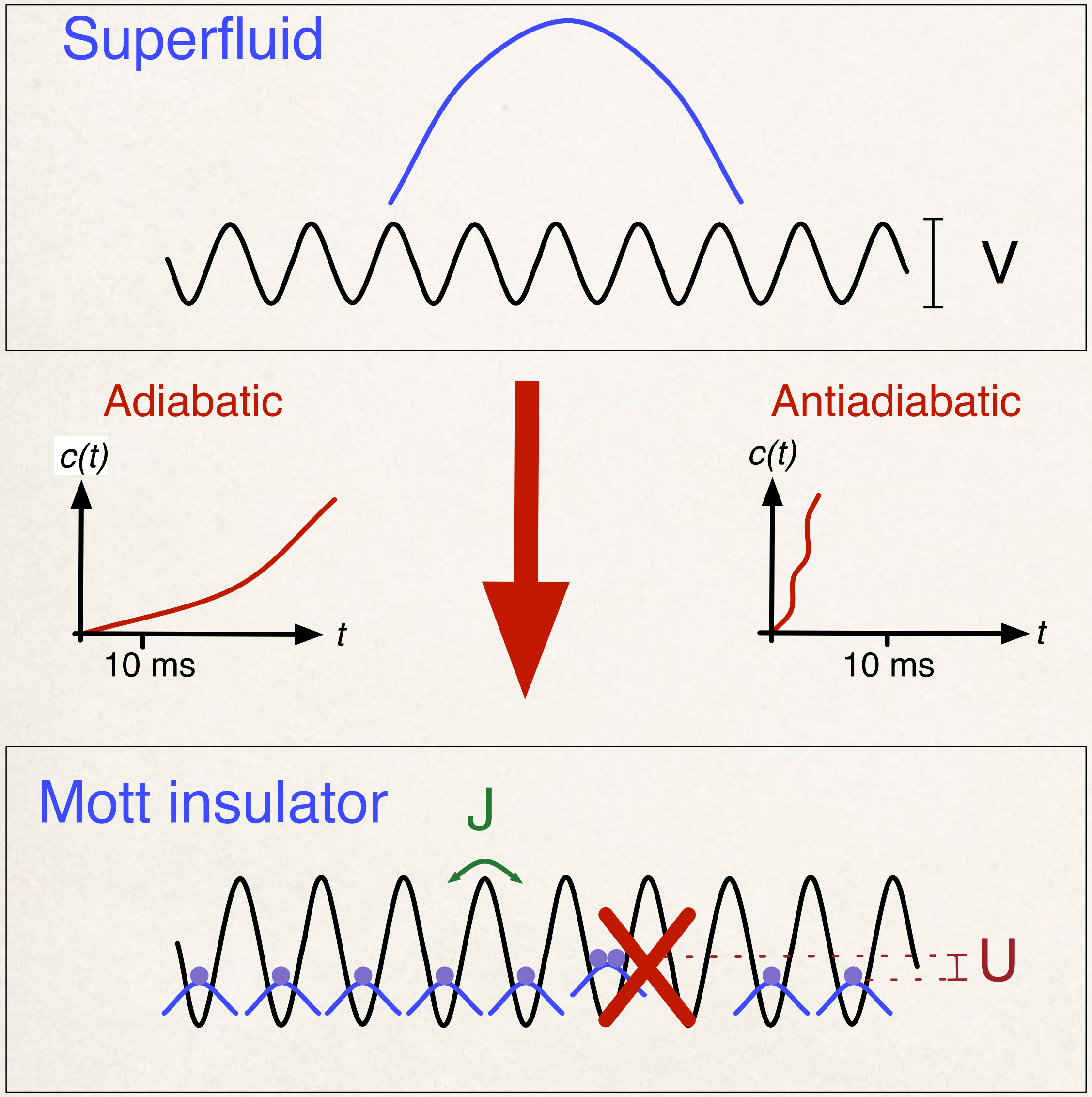
Optimal s: $I = \cos^2(s/2)$ | π | $I = 0$ | ∞

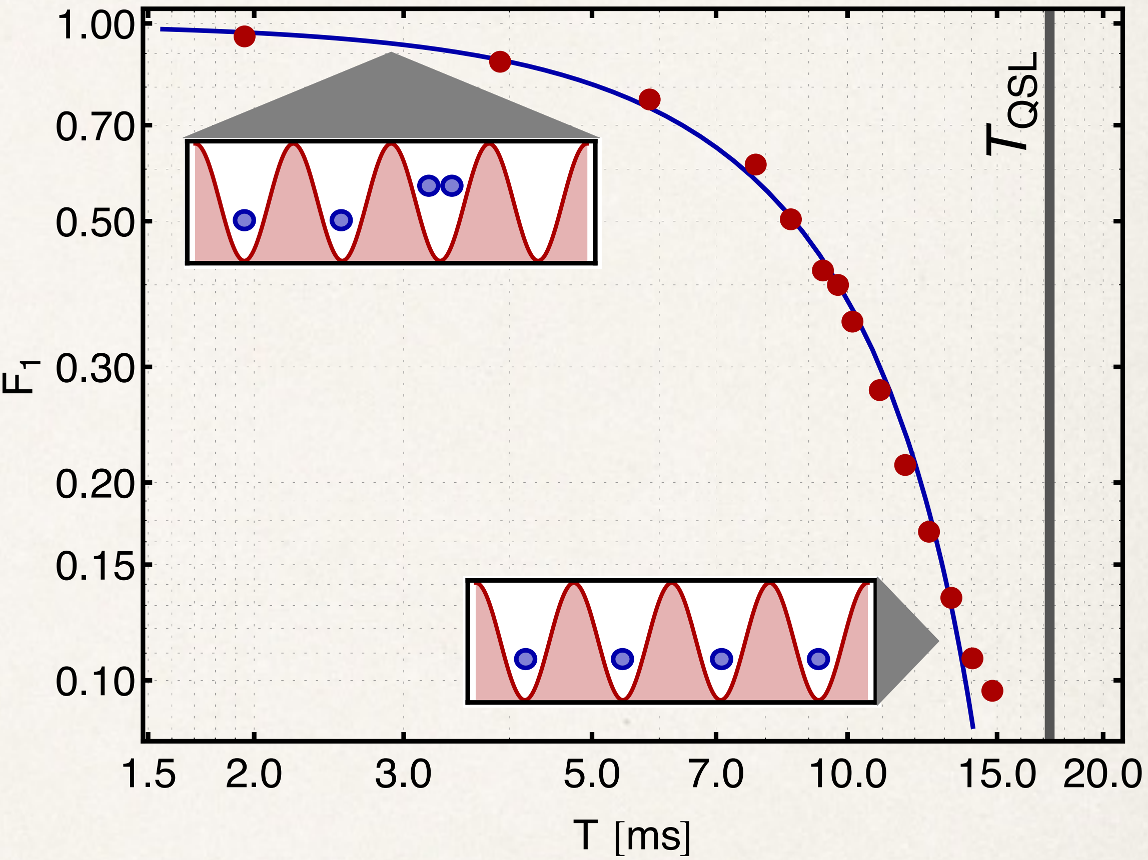
Kibble Zurek QSL Adiabatic

T. Caneva, T. Calarco, R. Fazio, G. E. Santoro, and SM Phys. Rev. A 84, 012312 (2011)

Open loop optimization



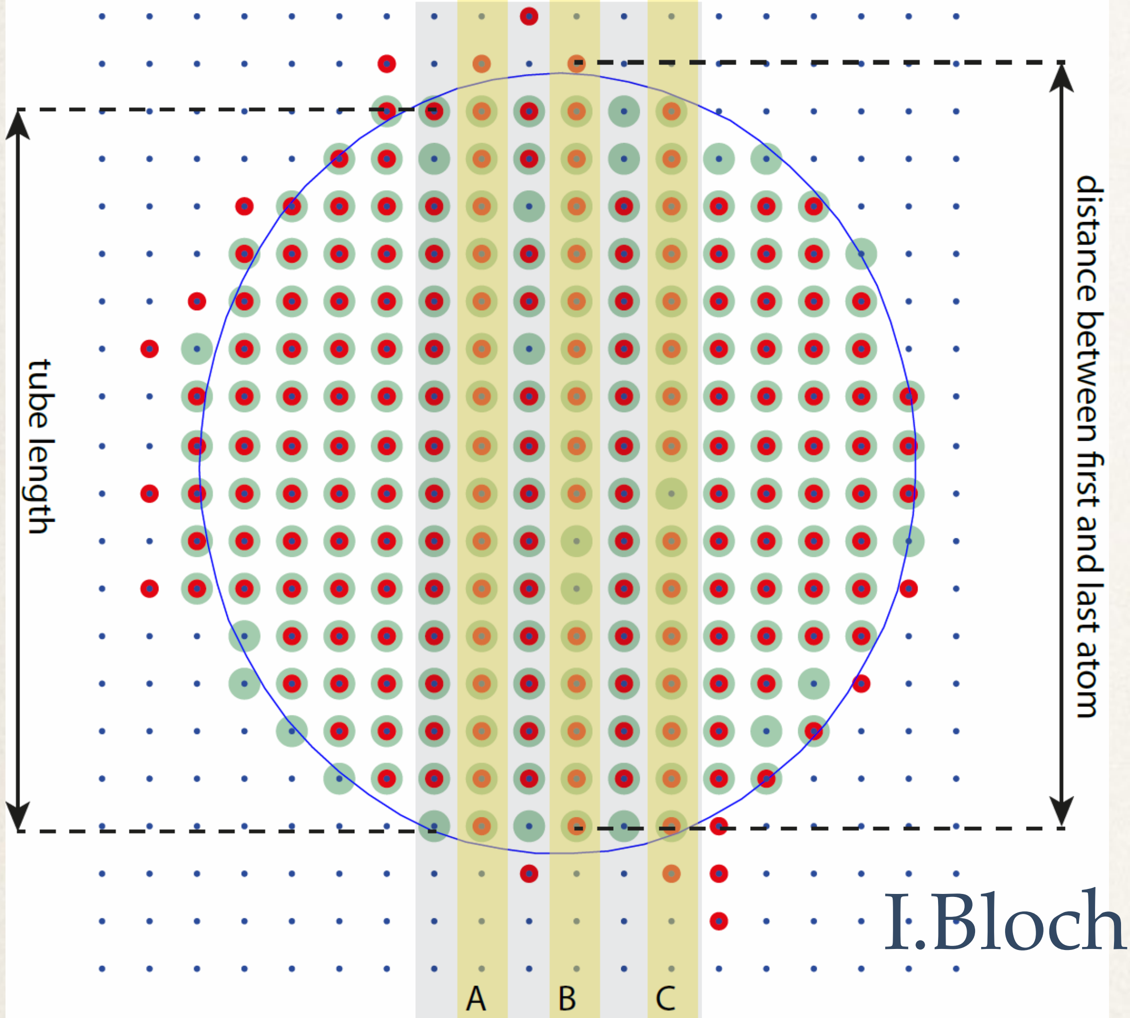




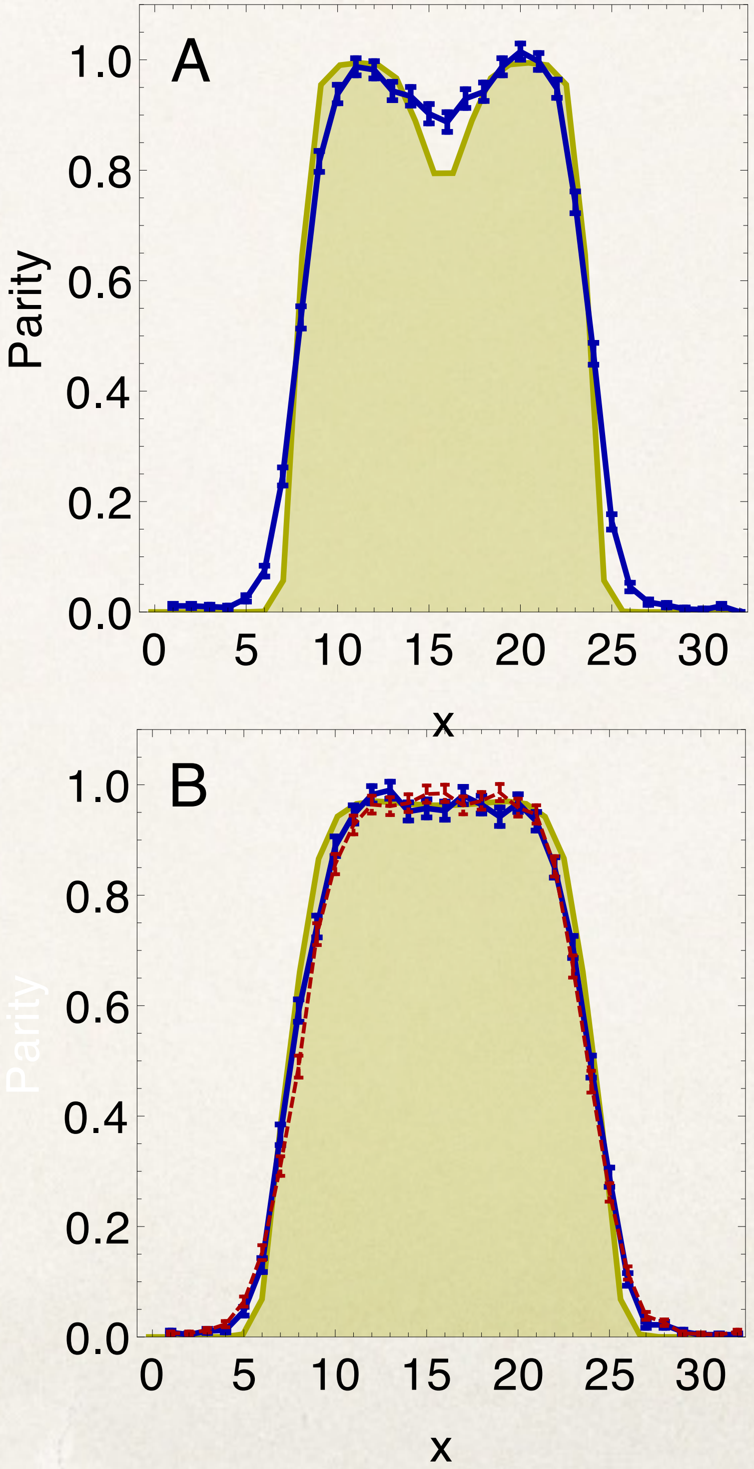
Quantum speed limit

$$T_{QSL} \sim 11 - 15\, ms$$

QPT crossing at QSL



I.Bloch's group



Linear ramp
T=12ms

Optimal ramp
T=12ms

Adiabatic ramp
T=110ms

T. Pichler, et. al. in preparation

Optimal control limits

- What are the physical limits of control of MBQS?

Controllability, Reachability, Quantum Speed Limit…

Control complexity

- What are the physical limits of control of MBQS?

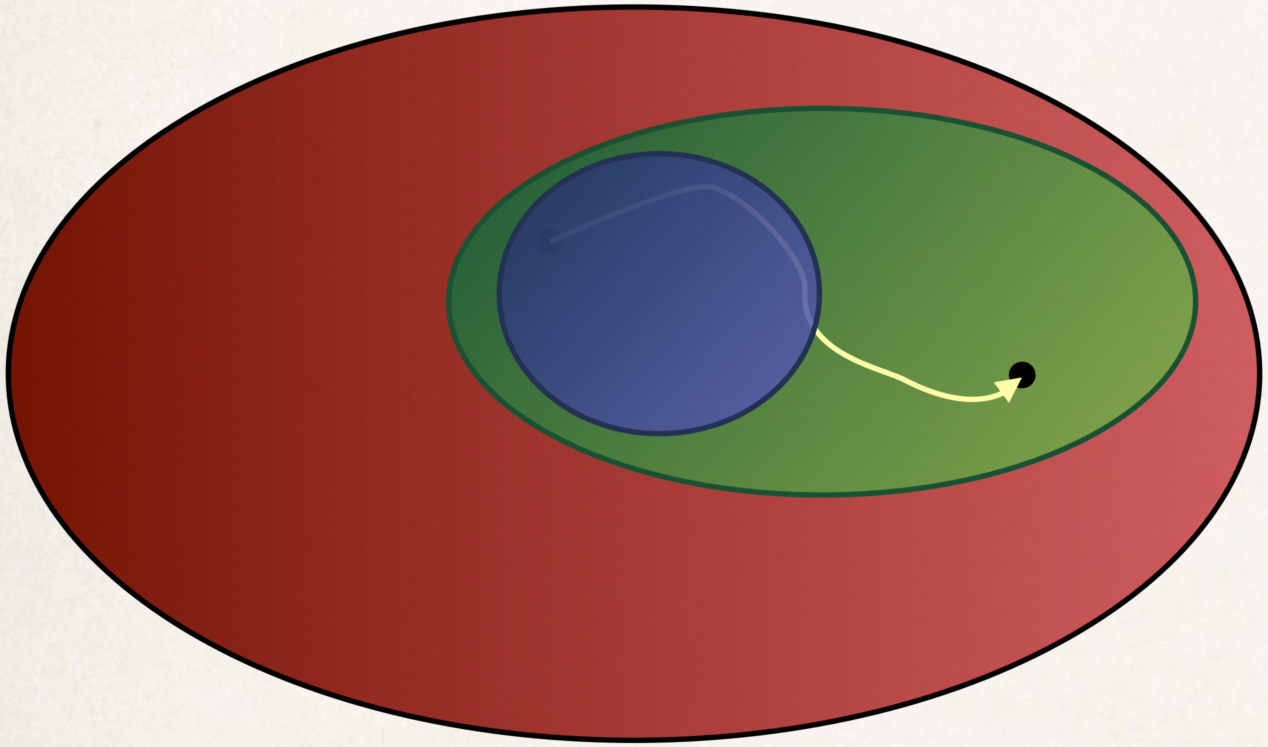
  Controllability, Reachability, Quantum Speed Limit, ...

- Are there any algorithmic/informational limits?

- How to characterize the complexity of the optimization task?

T. Caneva, A. Silva, R. Fazio, S. Lloyd, T. Calarco, S. Montangero, PRA (2014)

Reachable states



$$\mathcal{H} = \mathbb{C}^D$$

Reachable states $\mathcal{W}$

$$||H_D|| = ||H_C|| = 1$$

$$\gamma(t) \in [0 : 1] \forall t$$

$$H = H_D + \gamma(t) H_C$$

$$\dot{\rho} = \mathcal{L}(\rho, \gamma(t))$$

Time-poly Reachable states $\mathcal{W}^+$

$$D_{\mathcal{W}^+}(N)$$

Optimal control complexity

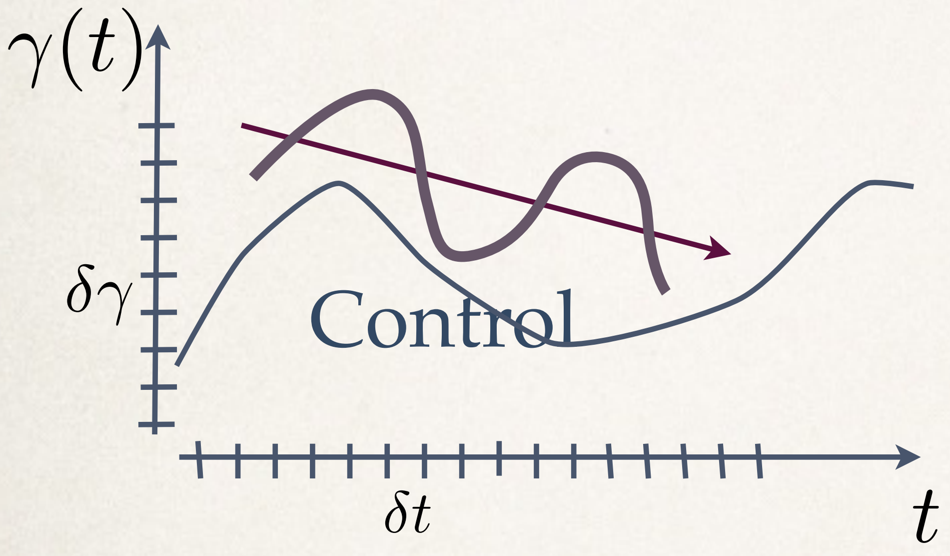
The (smoothed) complexity of an optimal control problem
is **polynomial** in the dimension
of the set of Time-poly reachable states $D_{\mathcal{W}^+}(N)$

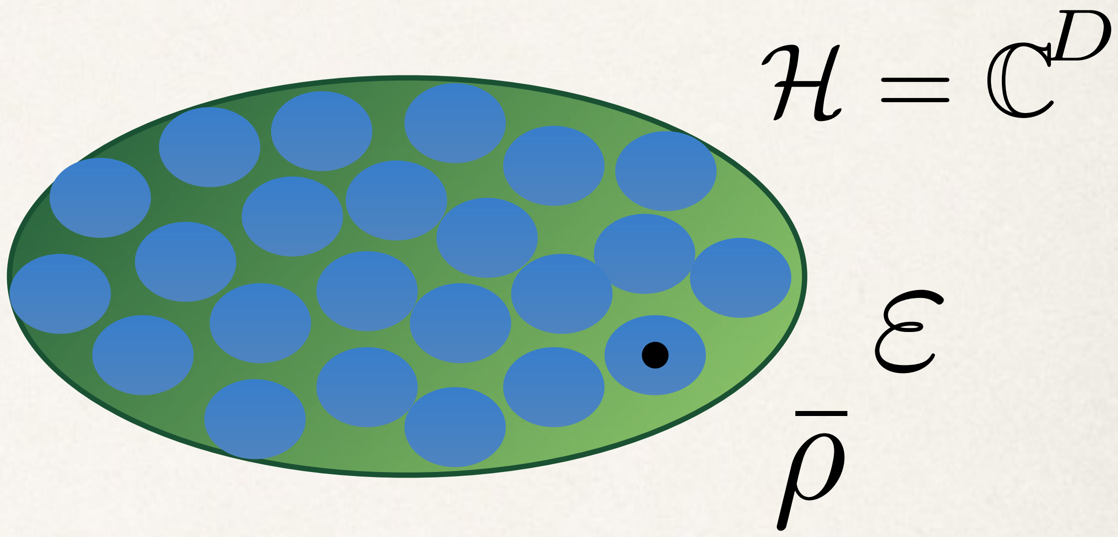
$\Rightarrow$

The optimal control problem can be solved efficiently via CRAB

S.Lloyd, S.Montangero, PRL (2014)

Lower Bound





Carried bits:

$$b_\gamma = T\,\Delta\Omega\,\kappa_S$$

Minimal needed bits:

$$b_S^- = \log \varepsilon^{-D}$$

$$b_\gamma \geq b_S^-$$

$$\varepsilon \geq 2^{-\frac{\kappa n_S}{D}}$$

$$n_s \geq D$$

Similar arguments lead to a polynomial upper bound…

$$n_s = Poly(D_{\mathcal{W}^+})$$

CRAB smoothed complexity

The simplex algorithm minimize the cost function

$$F(n_s) = F(Poly(D_{\mathcal{W}^+}(N)))$$

its smoothed complexity is polynomial in $D_{\mathcal{W}^+}$

$\Rightarrow$ Integrable and TN-simulatable dynamics can be efficiently controllable.

$\Rightarrow$ General non-Integrable systems are exponentially difficult to control.

Optimal control complexity

- t-DMRG allows to simulate dynamics of many-body quantum systems that can be described efficiently by a MPS state.
- A time-dependent MPS lives in a space which is Poly(n) T, thus the optimal control problem dimension is at most polynomial in n.
- Slightly entangled dynamics can be efficiently represented via MPS, thus can be also controlled.
- In general, what can be simulated / represented efficiently can also be optimally controlled!

Time bounds

Lower bound:

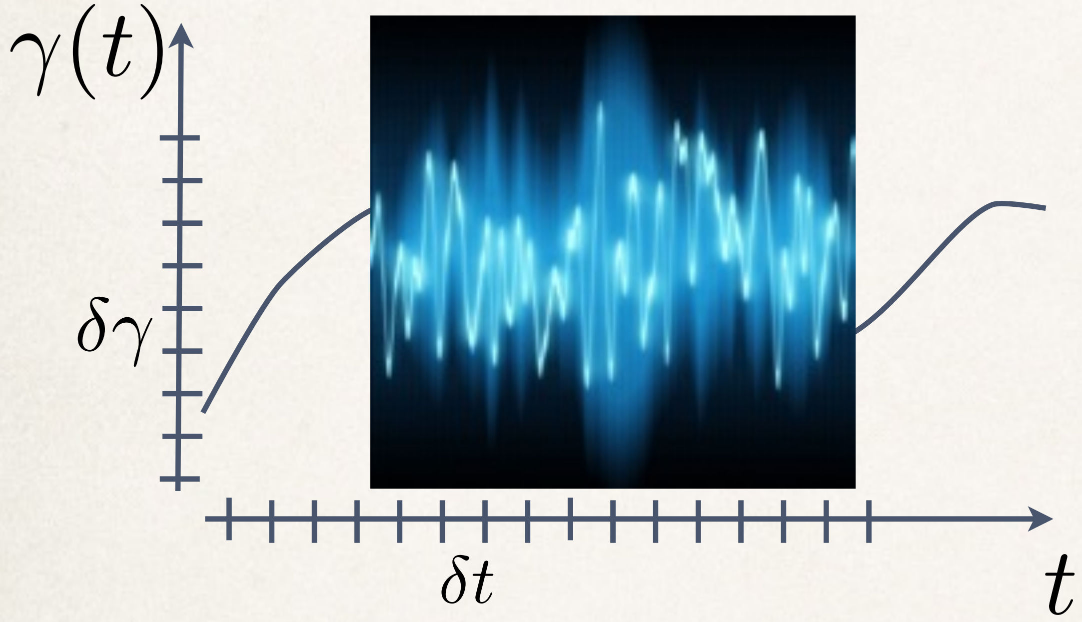
$$\varepsilon \geq 2^{-\frac{T\,\Delta\Omega\,\kappa_S}{D_{\mathcal{W}}+}} \quad \Longrightarrow \quad T \geq \frac{D_{\mathcal{W}}}{\Delta\Omega\,\kappa_S} \log(1/\varepsilon)$$

$$\Big\Downarrow \; \kappa_\varepsilon = \kappa_s$$

$$T \geq \frac{D_{\mathcal{W}}}{\Delta\Omega}$$

Continuos-time
Solovay-Kitaev theorem

Noise effects



$$\xcancel{\kappa_S = \log(1 + \gamma_{max}/\delta\gamma)}$$

bits per unit time

$$\varepsilon \geq (1 + S/N)^{-\frac{n_s}{D_{\mathcal{W}}}}$$

$$\varepsilon \gtrsim (N/S)^{n_s/D_{\mathcal{W}}}$$

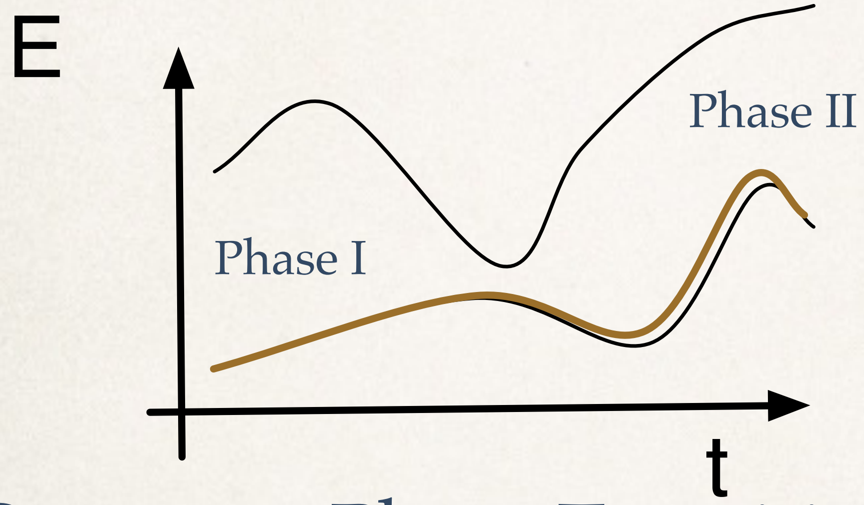
$$\Longleftarrow$$

Shannon-Hartley theorem

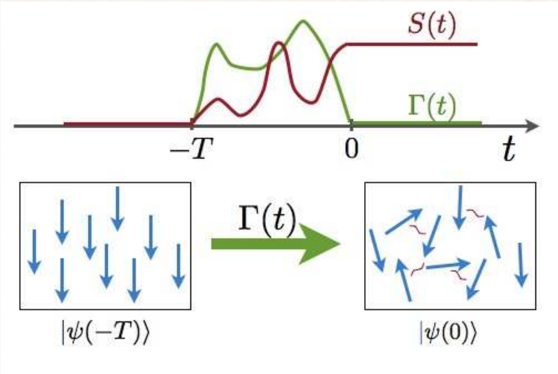
$$k_S = \log(1 + S/N)$$

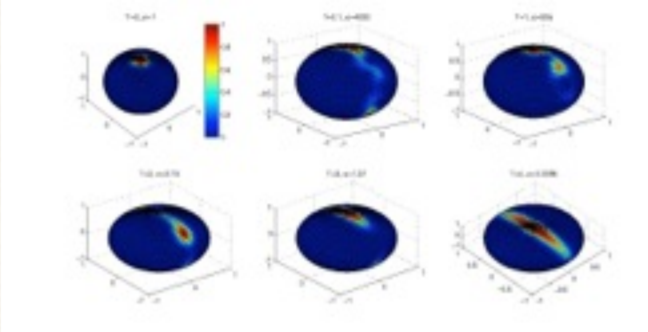
If $n_s = D_{\mathcal{W}}$ linearly sensitive to noise!

Applications (continued)



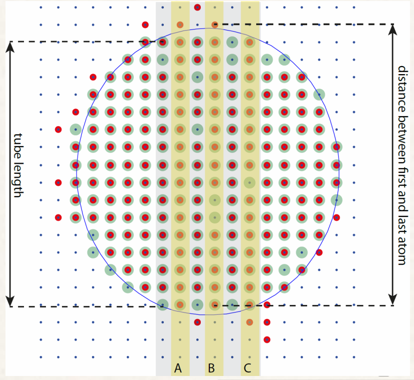
Quantum Phase Transition dynamics

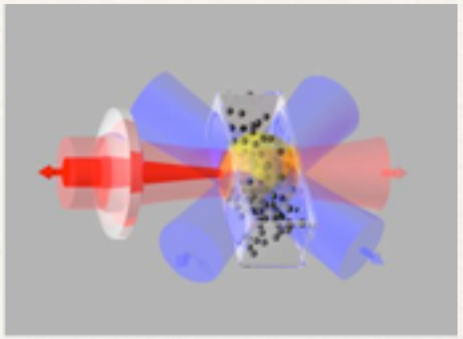


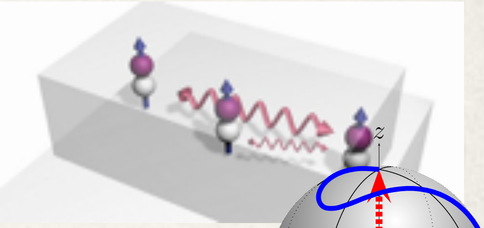


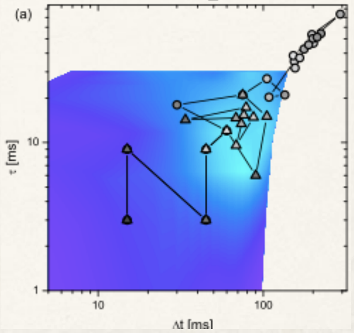
Entanglement/Squeezing manipulation

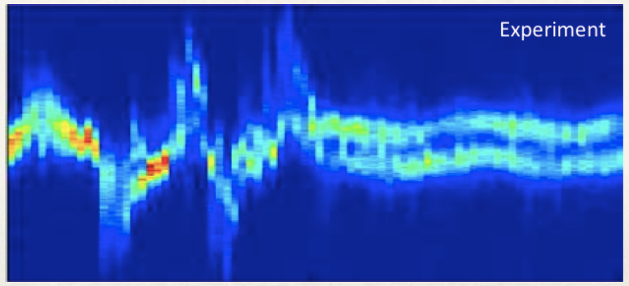
Light-harvesting dynamics

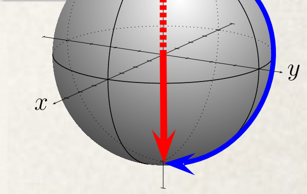






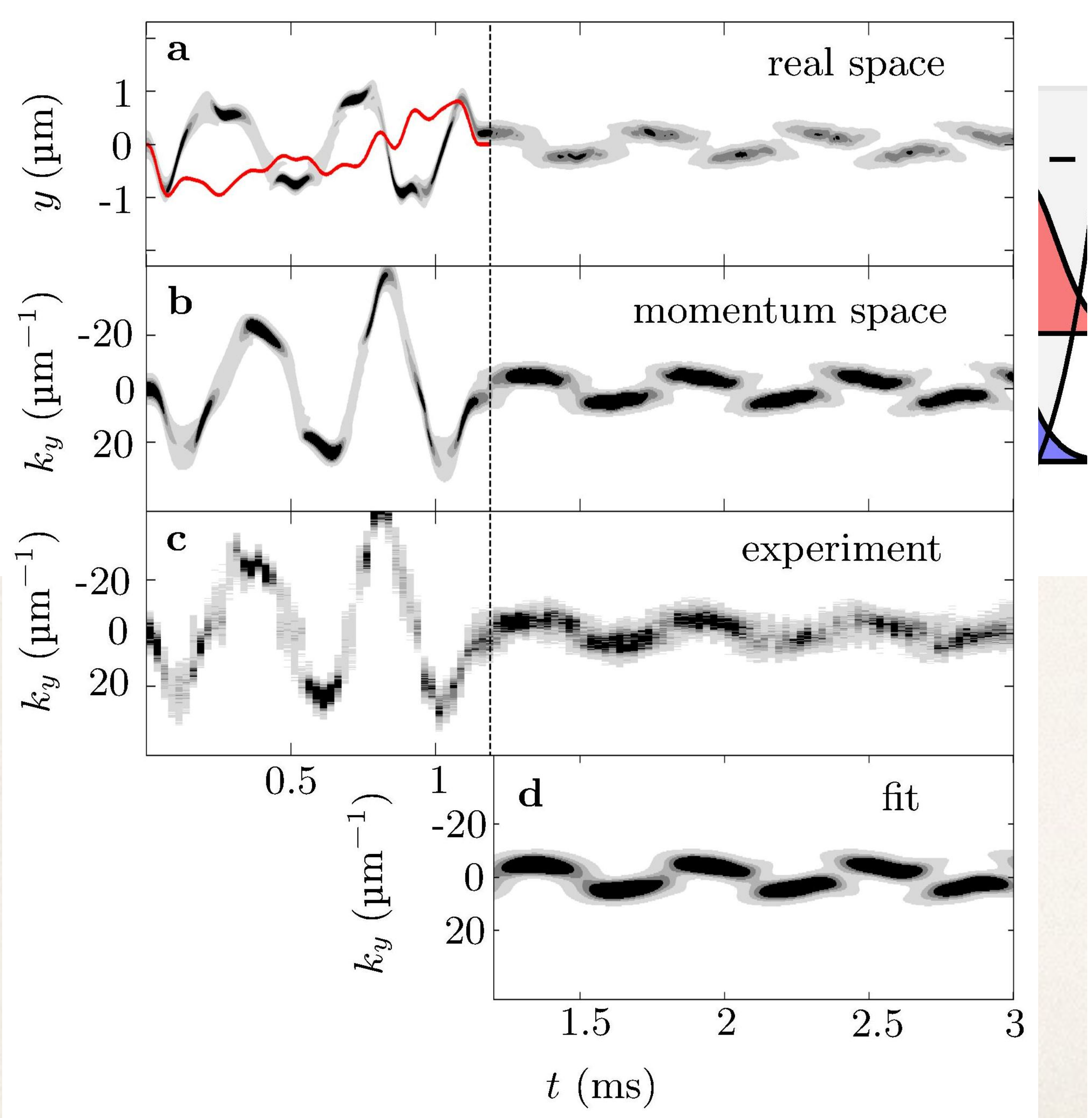


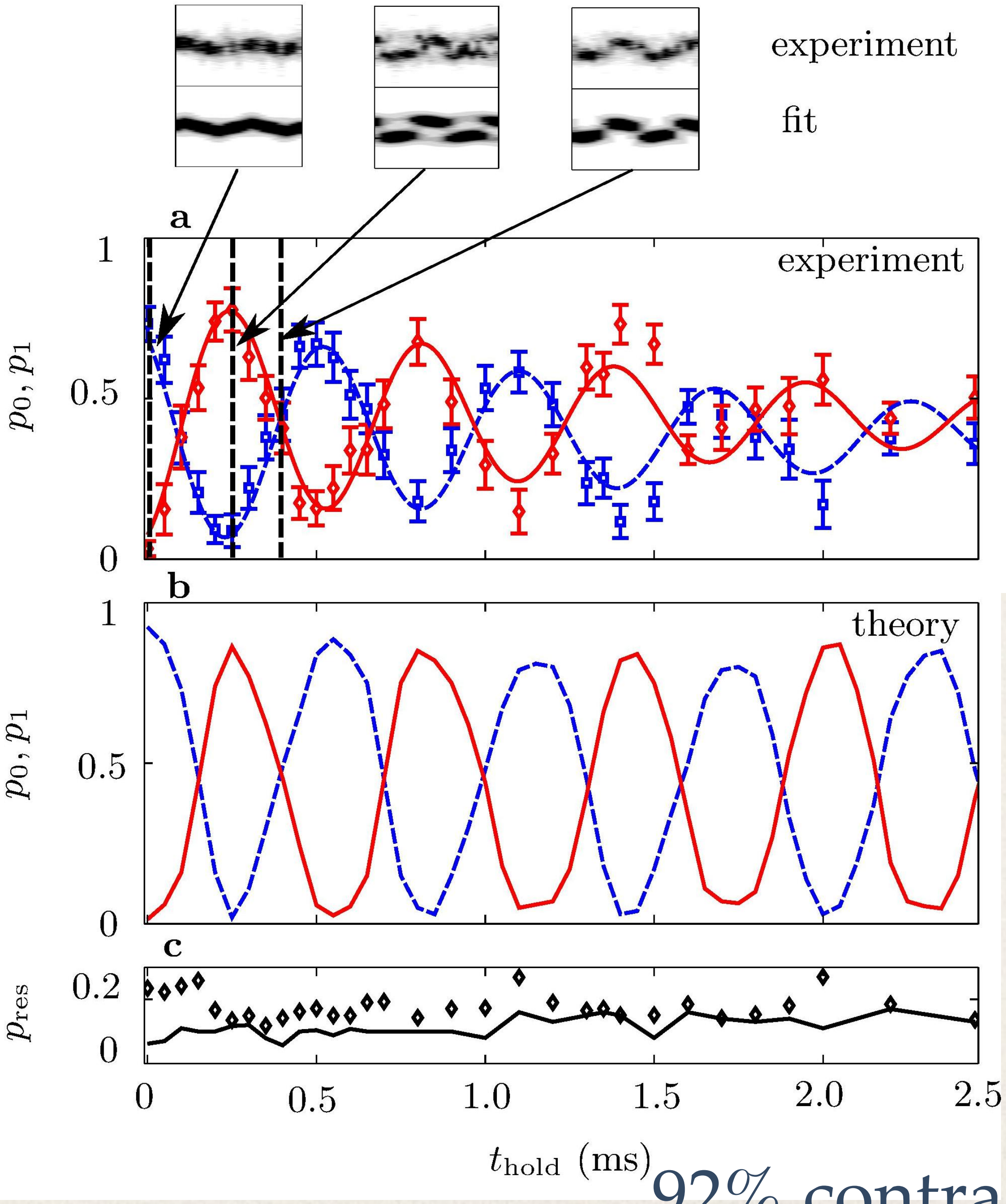




Optimal experimental protocols

Atom chip interferometer at QSL

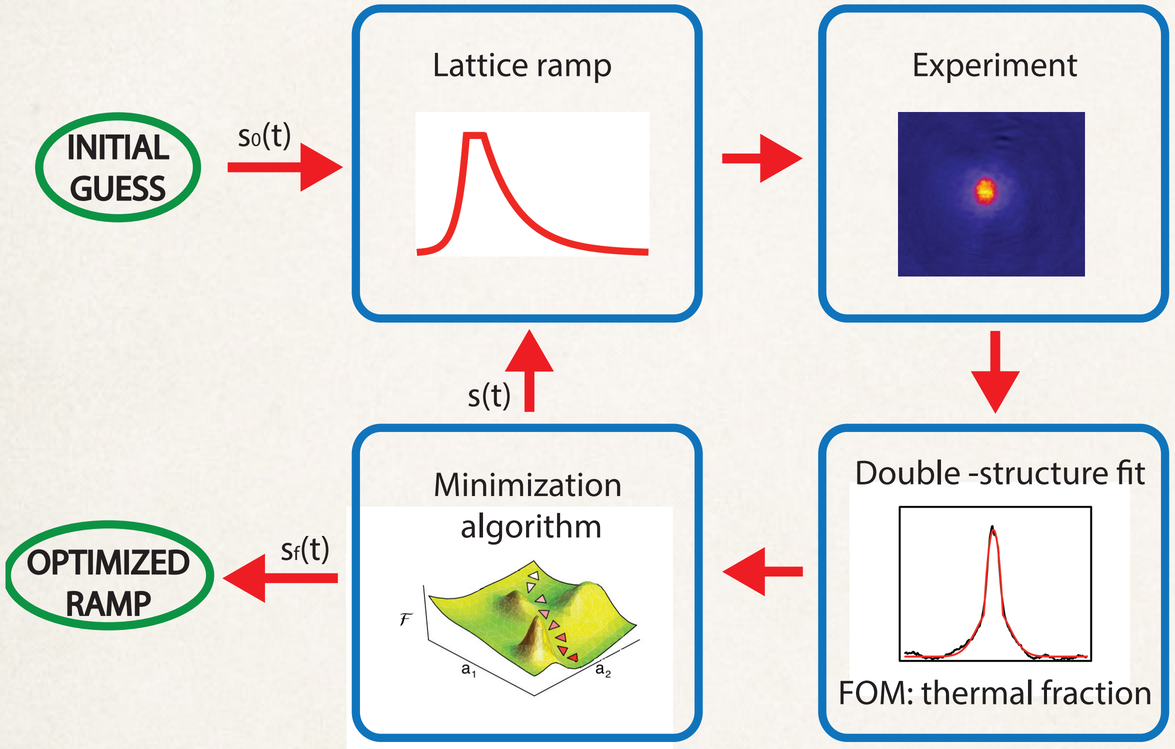




92% contrast

S. Van Frank, et.al. Nat. Comm. (2014)

Closed loop optimization



INITIAL GUESS

$s_0(t)$

Lattice ramp

Experiment

Double -structure fit

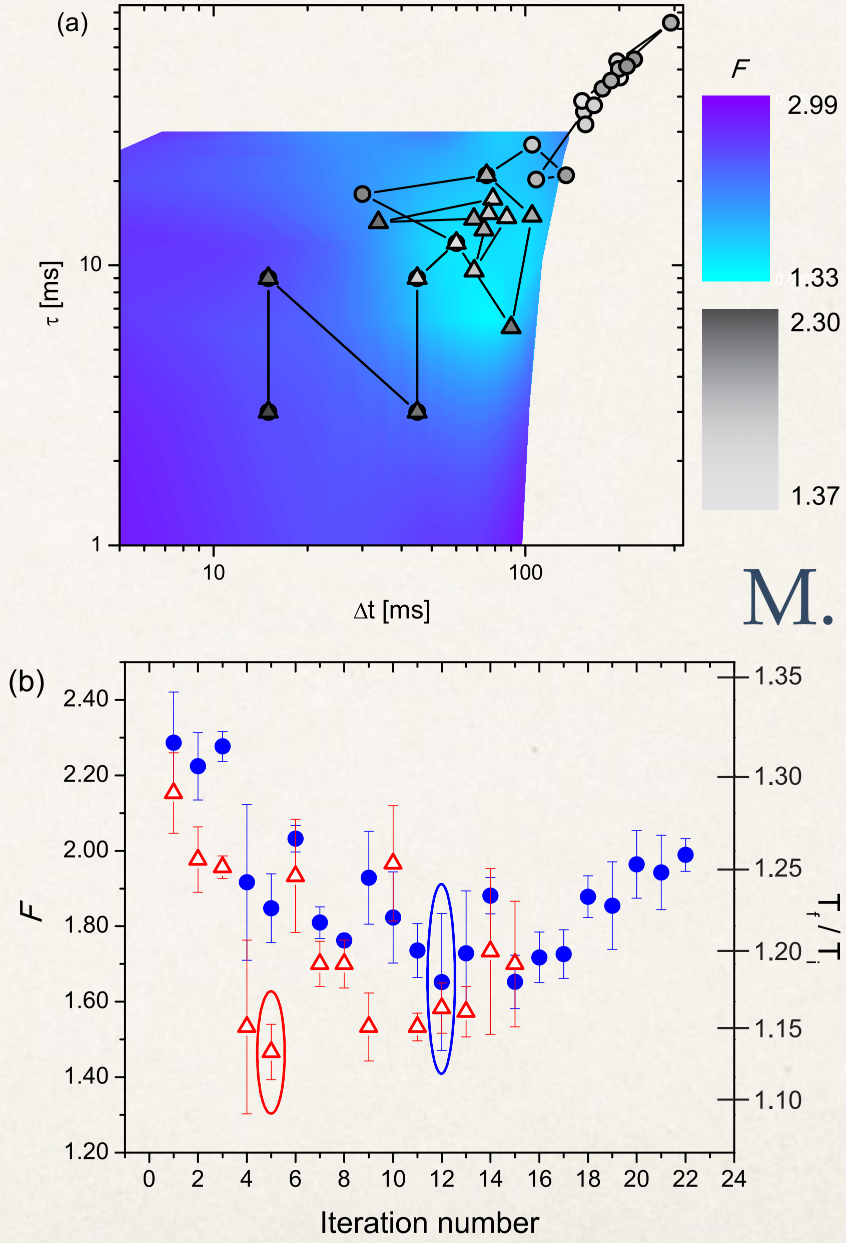
FOM: thermal fraction

Minimization algorithm

s(t)

$s_f(t)$

OPTIMIZED RAMP



M. Inguscio group

Run	Δt_{opt}	τ_{opt}	F_{uncorr}	Best F_{opt}
1	154 ms	35 ms	2.30 ± 0.03	1.73 ± 0.02
2	45 ms	9 ms	2.30 ± 0.03	1.40 ± 0.06

3D-1D crossover and QPT

$$T_{opt} \sim T_{ad}/3$$

$$FOM_{opt} \sim 0.9\ FOM_{ad}$$

S. Rosi, et. al. PRA 2013

Conclusions

- CRAB optimization can be applied successfully to MBQS dynamics opening new perspectives.
- What can be simulated can be controlled
- Optimal trajectories are robust with respect to noise and perturbations.
- Non-integrable MBQS are exponentially complex to be optimized (in general) but particular interesting protocols can be efficiently simulated and thus controlled.

Cartagena (Spain) 25-30/5/2015

New Trends in Complex Quantum systems dynamics - 2nd Edition

Thank you for your attention!

Tommaso Calarco
Tommaso Caneva
Matthias Mueller
Thomas Pichler
Enrique Rico
Martin Plenio
Fedor Jelezko
Boris Nayedov

Misha Lukin

Tilman Pfau
Robert Löwe

S. Lloyd

Rosario Fazio

Massimo Inguscio
Leonardo Fallani
Alain Bernard
Chiara Fort
Nicole Fabbri
Filippo Caruso

Immanuel Bloch
Marc Cheneau
Sebastian Hild

Peter Zoller
Marcello Dalmonte

Jörg Schmiedmayer
Thorsten Schumm
Sandrine van Frank
Wolfgang Rohringer

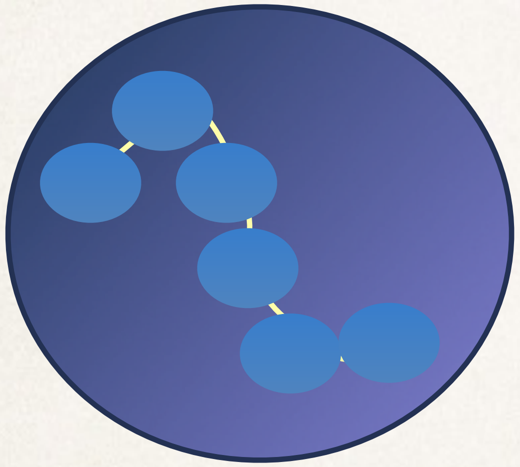
Funds:
SFB/TRR21 Co.Co.Mat.

IP-SIQS
IP-DIADEMS
STREP-DIAMANT
STREP-PICC
STREP-MALICIA

Numerics:
BW-Grid

www.dmrg.it

Upper-bound



$$n_\varepsilon = L/\varepsilon \leq T v_{max}/\varepsilon \;\; = Poly(D_{\mathcal{W}^+}) v_{max}/\varepsilon$$

Bounded energy T-poly

$$b_S^+ = \frac{Poly(D_{\mathcal{W}^+}) v_{max}}{\varepsilon} \log \varepsilon^{-D_{\mathcal{W}^+}}$$

$$b_\gamma \leq b_S^+$$

$$Poly'(D_{\mathcal{W}^+}) v_{max}/\varepsilon \geq n_s$$

In conclusion:

$$n_s = Poly(D_{\mathcal{W}^+})$$

Smoothed complexity

Analysis of algorithmic complexity **under perturbed input**

Machine learning, Numerical analysis, Discrete mathematics...

Linear programming:

$$\begin{aligned} \max c^T x \\ \text{subject to} \\ Ax \leq b \end{aligned}$$

Simplex algorithm has polynomial smoothed complexity!