Cooling of many-body systems through optimal control of quantum evolution

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Outline

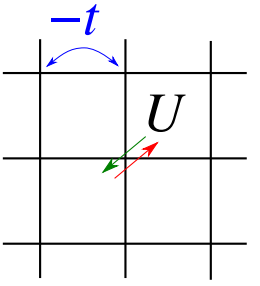
- Quantum simulations through cooling
- The confines of the (second) law
- The case of two coupled quasicondensates
- Challenge of a universal scheme

Quantum simulations through cooling

Major bottleneck in condensed matter physics:
Solving interacting model Hamiltonians.

material

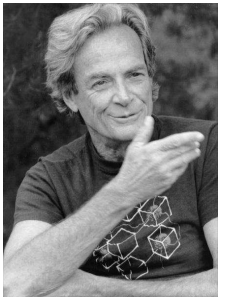
approximate
abstraction



model

limited success

solution

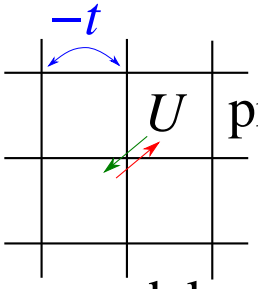


"I therefore believe it's true that with a suitable class of quantum machines you could imitate any quantum system, including the physical world."
-Richard P. Feynman 1982

Quantum simulations through cooling

Two developments with cold atoms:
1) Tuning of interactions with Feshbach resonances (1998)
2) Creation of periodic optical potentials in various dimensions (2002)

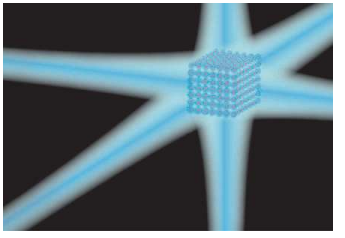
Atomic physicists can create many-body Hamiltonians



model

precise and tunable

Science, 320, 312 (2008)

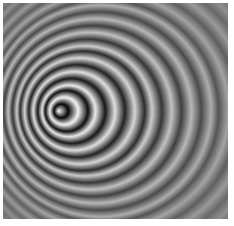


artificial material

- create Hamiltonian
- cool it to near $T = 0$
- measure observables

Quantum simulations through cooling

Laser and evaporative cooling do an impressive job but have limitations.



Important properties:

- Thermal isolation
- Time-dependent tunability

These properties make unitary evolution a potential resource for transforming states.

Is it useful for cooling further?

The confines of the (second) law

"It is imposible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects." Lord Kelvin

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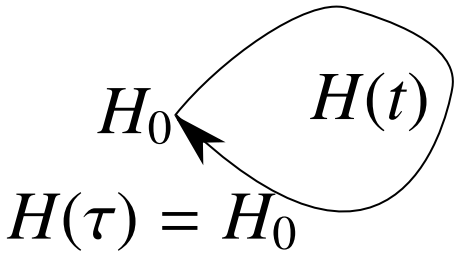
cyclic process



Can tr $[H_0\rho(\tau)] < \mathrm{tr}(H_0\rho_0)$?

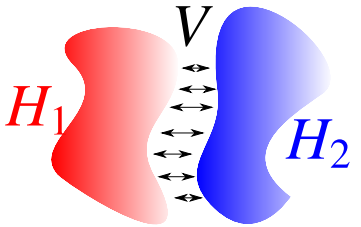
No for thermal ρ_0.

Can be proved rigorously using a celebrated theorem of discrete math: Birkhoff-von Neumann theorem.

The confines of the (second) law

If we are interested in cooling a system with a Hamiltonian H_0, we need to start from a different Hamiltonian.

One strategy:



$$H = H_1 + H_2 + V(\{\lambda\})$$

$$V(t=0) = V_0$$
$$V(t=\tau) = 0$$

Assumptions:

i) We can tune $\{\lambda(t)\}$ in a given range

ii) We can cool the system to $1/\beta_0$ with current methods.

iii) We have a time τ to carry out a unitary process.

Goal:

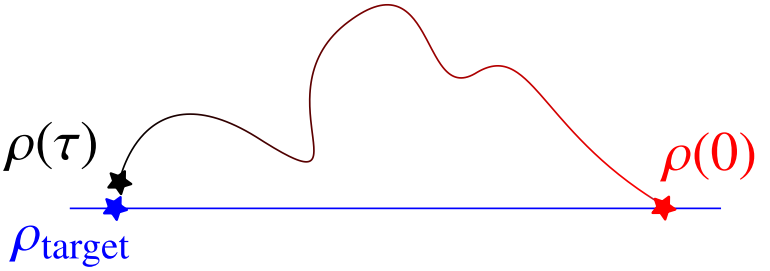
Find the optimal $\lambda(t)$ so that $\mathcal{E} = \mathrm{tr}\,[H_1\rho(\tau)]$ is minimized.

The confines of the (second) law

This is a problem in optimal control.

given $\rho(0)$ and protocol $\lambda(t)$, $0 < t < \tau$ $\longrightarrow$ unique $\rho(\tau)$

Find the protocol $\lambda(t)$ that minimizes a cost function $\mathcal{E}[\rho(\tau)]$



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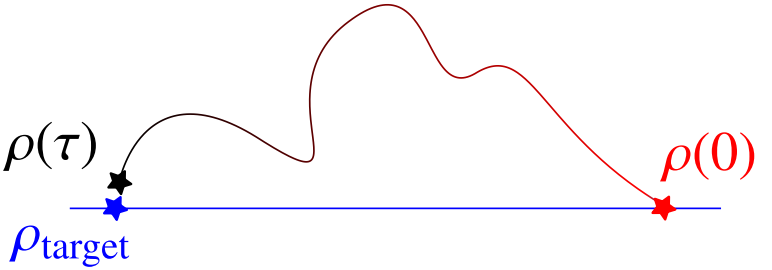
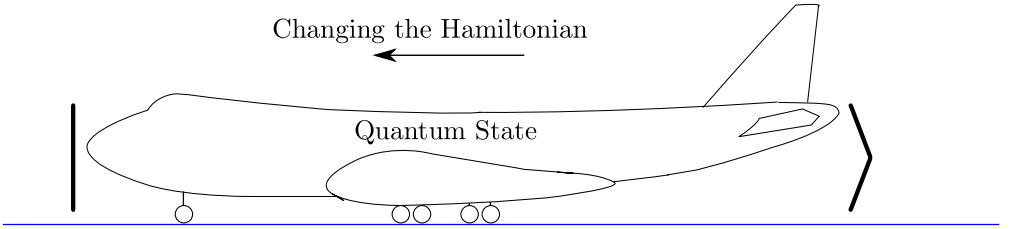
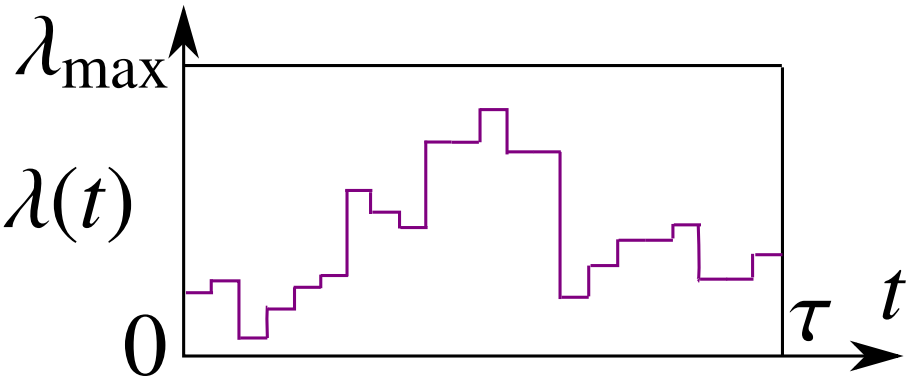


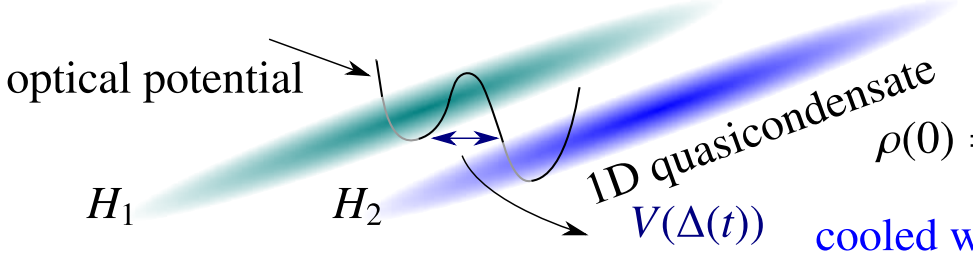


The confines of the (second) law

If we know $\mathcal{E}$ for arbitrary $\lambda(t)$, we can do straightforward simulated annealing.



The case of two coupled quasicondensates



$$\rho(0) = \exp\left[-\beta_0 (H_1 + H_2 + V(\Delta_0))\right]$$

cooled with current state of the art

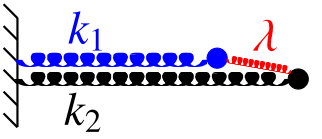
$$H_i = \frac{v_i}{2} \int dx \left[\frac{\pi}{g_i} \Pi_i^2(x) + \frac{g_i}{\pi} \left(\partial_x \Phi_i(x)\right)^2 \right]$$

$$V = -2\frac{\Delta}{a} \int dx \cos\left[\Phi_1(x) - \Phi_2(x)\right]$$

Harmonic approximation:

$$\begin{aligned} H &= \sum_i \sum_{q>0} \left[\frac{v_i \pi}{4 g_i} \left(\Pi_q^{\Re i}\right)^2 + \frac{v_i g_i}{\pi} q^2 \left(\Phi_q^{\Re i}\right)^2 \right] \\ &+ \sum_{q>0} 2\Delta \left(\Phi_q^{\Re 1} - \Phi_q^{\Re 2}\right)^2 + \Re \leftrightarrow \Im, \end{aligned}$$

The case of two coupled quasicondensates

Each mode: 

$$m_i = \frac{2g_i}{\pi v_i}, \qquad k_i = \frac{2}{\pi} v_i g_i q^2, \qquad \lambda = \frac{4\Delta}{a}.$$

$$K(\lambda) = \begin{pmatrix} (k_1+\lambda)/m_1 & -\lambda/\sqrt{m_1 m_2} \\ -\lambda/\sqrt{m_1 m_2} & (k_2+\lambda)/m_2 \end{pmatrix}.$$

normal-mode frequencies $\bar{\omega}_1$ and $\bar{\omega}_2$ $\quad K(\lambda) = Q(\lambda)\,\mathrm{diag}(\bar{\omega}_1^2, \bar{\omega}_2^2)\,Q^T(\lambda)$

$$\rho_0 = \frac{1}{\mathcal{Z}} e^{-\beta_0 \bar{\omega}_1(\lambda_0)\bar{a}_1^\dagger(\lambda_0)\bar{a}_1(\lambda_0)} e^{-\beta_0 \bar{\omega}_2(\lambda_0)\bar{a}_2^\dagger(\lambda_0)\bar{a}_2(\lambda_0)}$$

$$a_1(t) = \sum_i \left[u_i(t)\bar{a}_i(\lambda_0) + v_i(t)\bar{a}_i^\dagger(\lambda_0) \right]$$

dynamical variables with simple initial conditions
and linear (in $\lambda(\tau - t)$) equations of motion

The case of two coupled quasicondensates

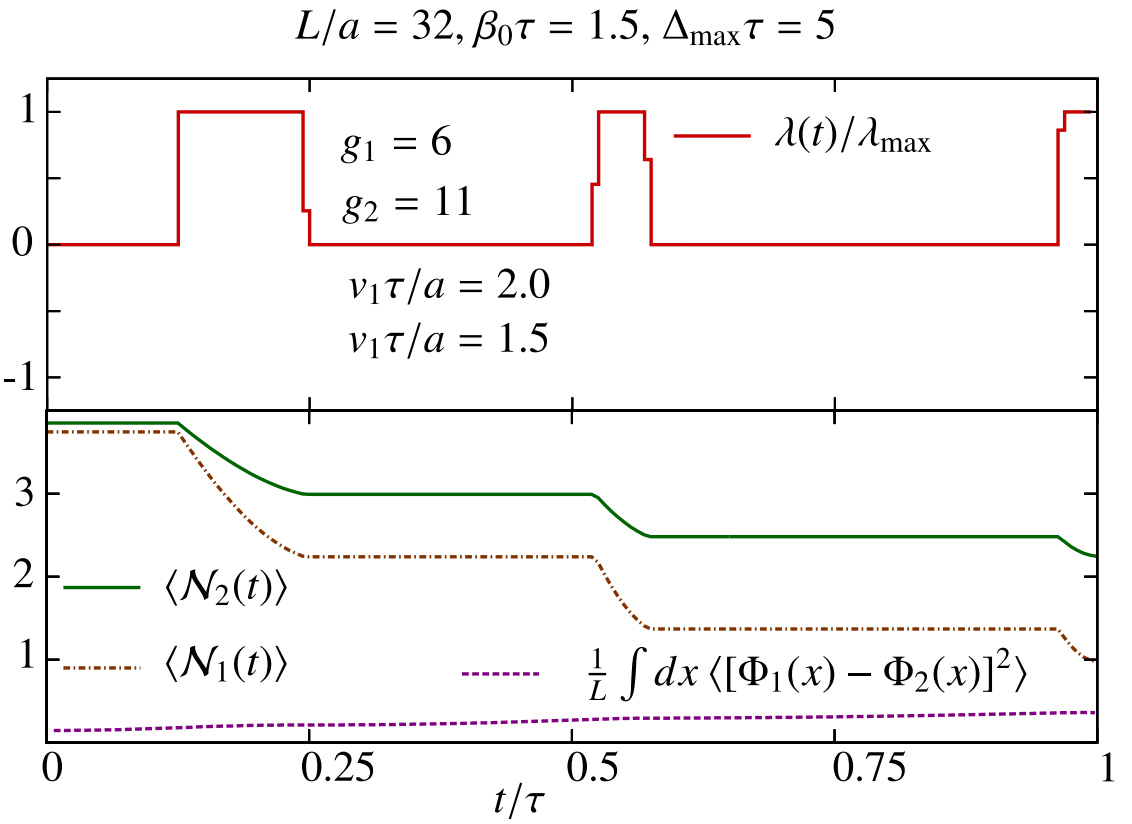
$$\langle n_1(t)\rangle = \mathrm{tr}\left[a_1^\dagger(t)a_1(t)\,\rho_0\right] = \sum_i |u_i(t)|^2\bar{n}_i(0) + |v_i(t)|^2(1+\bar{n}_i(0))$$

$$\bar{n}_i(0) \equiv \mathrm{tr}\left[\bar{a}_i(\lambda_0)^\dagger\bar{a}_i(\lambda_0)\,\rho_0\right] = \left(e^{\beta_0\bar{\omega}_i(\lambda_0)} - 1\right)^{-1}$$

From single-mode to many-mode:

$$\langle \mathcal{N}_1(t)\rangle = 2\sum_{0<q<\Lambda}\langle n_1^q(t)\rangle, \quad \langle \mathcal{E}_1(t)\rangle = 2v_1\sum_{0<q<\Lambda} q\,\langle n_1^q(t)\rangle$$

The case of two coupled quasicondensates



The case of two coupled quasicondensates

Bang-Bang expected from Pontryagin's maximum principle:

Dynamical variables	Equations of motion	Initial conditions
$\{x(t)\}$	$\dot{x}_j = f_j(\{x, \alpha\})$	$x_j(0) = x_j^0$

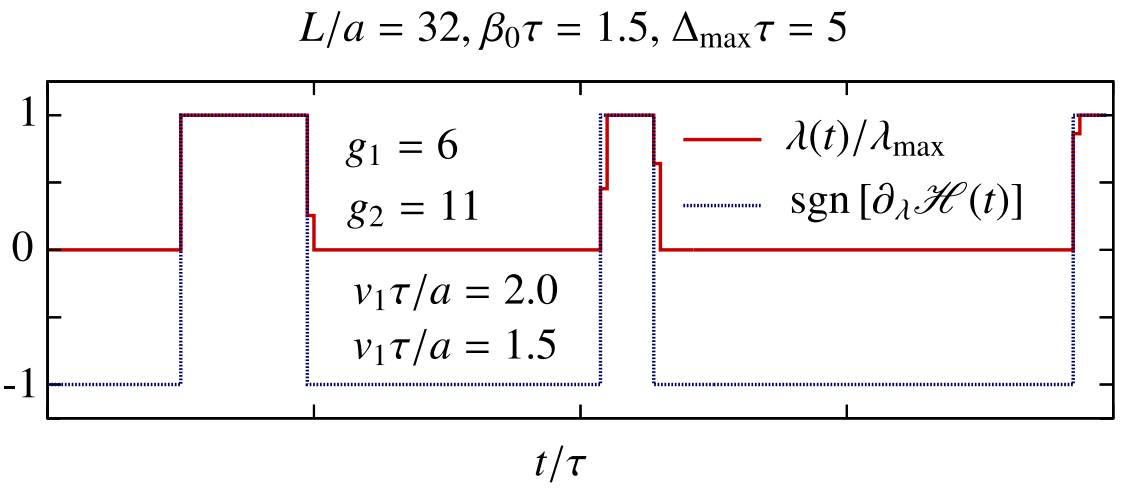
Maximize $g(\{x(\tau)\})$ over all admissible $\{\alpha(t)\}$.

$$\mathscr{H}(\{x, p, \alpha\}) = \sum_j p_j(t)\, f_j(\{x, \alpha\})$$

$$\mathscr{H}^* \equiv \mathscr{H}(\{x^*, p^*, \alpha^*\}) = \max_{\{\alpha\}} \mathscr{H}(\{x^*, p^*, \alpha\}),$$

The case of two coupled quasicondensates

Bang-Bang protocol determined by sgn($\partial_\lambda \mathscr{H}$)



The protocol removes energy from H_1 and H_2 and puts it in the coupling V. Should be generically possible in the absence of many-body localization. At the end, we decouple (set $V = 0$), and wait to thermalize.

Challenge of a universal scheme

We saw in a simple example that

1. Coupling a quantum system to a replica, and performing optimal control on the coupling constant can reduce its energy beyond the current state of the art.

2. The cooling is significant: 3-5 times with dimensionless parameters of order unity.

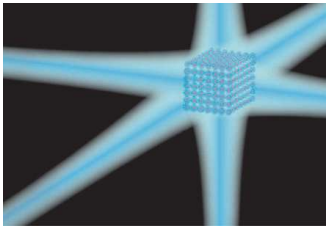
3. If we know the final energy for all allowed protocols, a simple generic MC algorithm can find the optimal protocol.

Challenge of a universal scheme

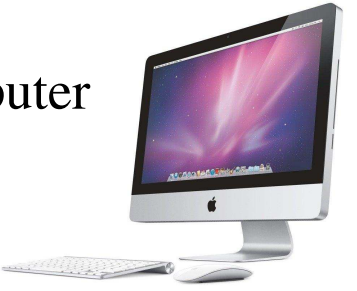
For complicated systems, it is not easy to compute the final energy for a given protocol.

How about letting the system itself do the MC?

prepare at β_0, evolve with a given protocol, measure the cost function



feed it into a classical computer



run a MC step, adjust the protocol, feed it back