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arXiv:1210.4323

Necessary and sufficient condition for quantum adiabatic evolution by unitary control fields

(another shortcut in 5 mins)

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Telluride, 2014

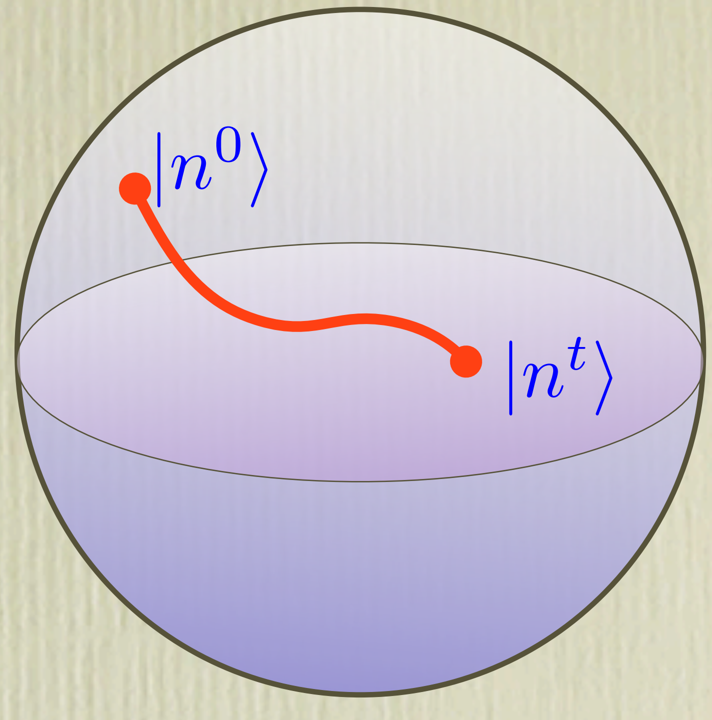
arXiv:1210.4323

Outline

- Introduction
- Gauge invariant formalism for adiabatic evolution
  - Error bounds
  - A necessary and sufficient condition
- Adiabatic evolution by pulse sequence
- Adiabatic evolution by continuously varying fields
- Marzlin-Sanders inconsistency (degenerate Hamiltonians)

Adiabatic theorem

If $H(t) = H(\mathbf{R}(t))$ is "slowly varying", "transitions" are suppressed.



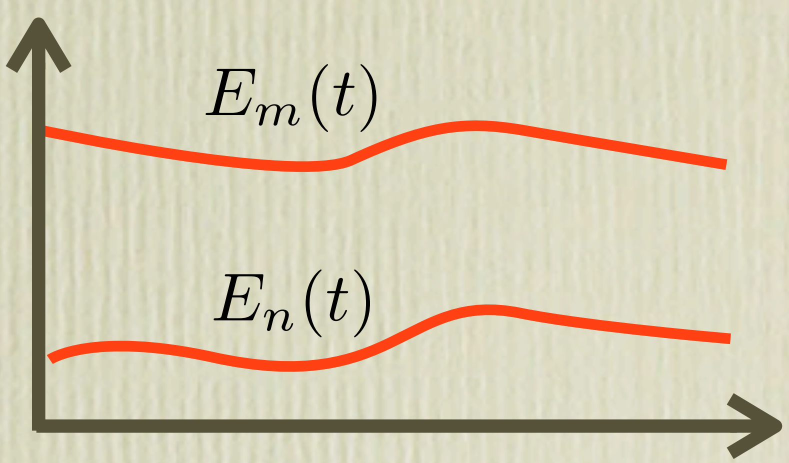
That is,

$$|n^0\rangle \to e^{i\phi}|n^t\rangle$$

when

$$\hbar \frac{d\mathbf{R}(t)}{dt} \ll |E_n(t) - E_m(t)|$$

[A. Messiah, Quantum Mechanics, Vol. II, (1965)]



What does "slowly varying" mean?
Can we have many energy crossings?

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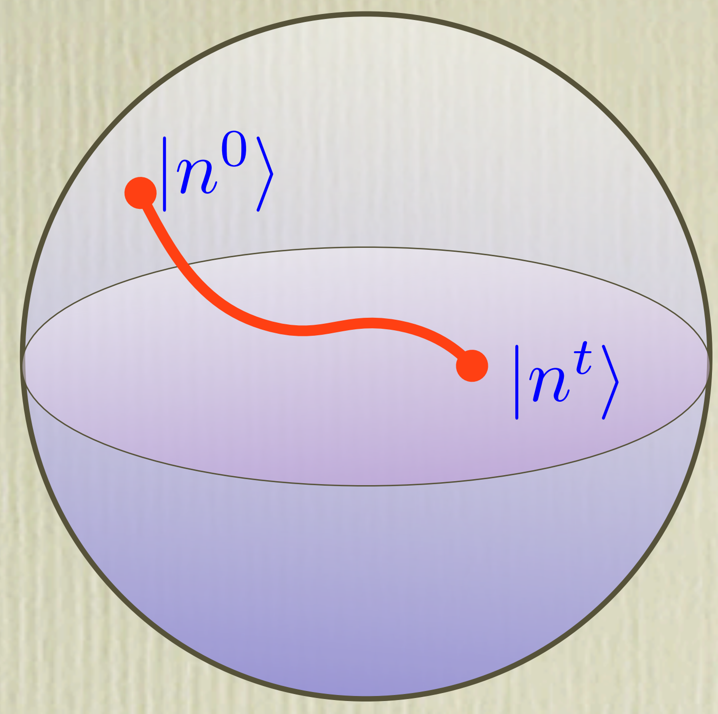
Geometric phase

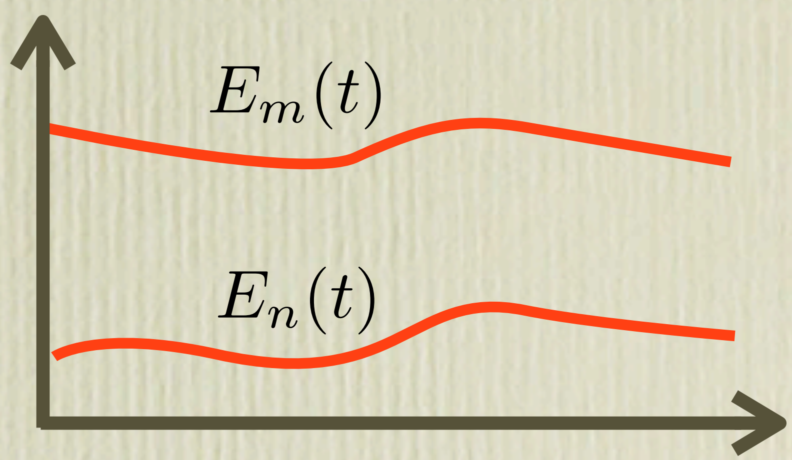
Adiabatic evolution: $|n^0\rangle \rightarrow e^{i\phi}|n^t\rangle$



$$\phi = \int_0^t E_n dt' - \int_0^t \langle n^{t'}|i\frac{d}{dt'}|n^{t'}\rangle dt'$$

Dynamic phase

Geometric phase
(Berry phase)



more general: non-Abelian

[M. Berry (1984); F. Wilczek & A. Zee (1984)]

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Nonadiabatic correction

The change of $H(t)$ can not be infinitely slow, and there is a nonadiabatic correction $U_{\mathrm{Dia}}(t)$.

Thus, the system evolution should be

$$U(t) \; = \; U_{\mathrm{Dyn}}(t) \, U_{\mathrm{Geo}}(t) \, U_{\mathrm{Dia}}(t).$$

Dynamic phase

Geometric phase

Nonadiabatic correction

Target: $U_{\mathrm{Dia}}(t) \rightarrow I$

Adiabatic evolution: $H(t)|n^t\rangle = E_n(t)|n^t\rangle$

Counterdiabatic driving: $[H(t) + H_c]|n^t\rangle \neq E_n(t)|n^t\rangle$

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Adiabatic condition

$$U(t) \;=\; U_{\rm Dyn}(t)\; U_{\rm Geo}(t)\; U_{\rm Dia}(t). \qquad\qquad U_{\rm Dia}(t) \to I$$

* A widely used quantitative condition:

$$\left|\frac{\langle n^t|\dot{m}^t\rangle}{E_n(t)-E_m(t)}\right| \ll 1$$

sufficient? No

[K.-P. Marzlin, B.C. Sanders, D. M. Tong, M. H. S. Amin, Daniel Lidar, ...]

necessary? Under debate

[D. M. Tong, M. Zhao, J. Wu, D. Comparat, ...]

* New proposed conditions

[Sergio Boixo, Daniel Lidar, Rolando Somma, D. Comparat, ...]

energy crossing?

[J. E. Avron & A. Elgart]

Need an exact $U_{\rm Dia}(t)$ (with possible degeneracy)

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Gauge invariant formalism

$$U(t) \;=\; U_{\mathrm{Dyn}}(t)\, U_{\mathrm{Geo}}(t)\, U_{\mathrm{Dia}}(t)$$

$$U_{\mathrm{Dyn}}(t) = \sum_{n,j} e^{-i\int_0^t E_n(t')dt'} |n_j^t\rangle\langle n_j^t|$$

$$U_{\mathrm{Geo}}(t) = \sum_{n,j} |n_j^{\boldsymbol{R}}\rangle\langle n_j^{\boldsymbol{R}_0}| \mathcal{P} e^{i\int_{\boldsymbol{R}_0}^{\boldsymbol{R}} \sum_{n,p,q} |n_p^{\boldsymbol{R}_0}\rangle\langle n_p^{\boldsymbol{R}'}| i\nabla_{\boldsymbol{R}'} |n_q^{\boldsymbol{R}'}\rangle\langle n_q^{\boldsymbol{R}_0}| \cdot d\boldsymbol{R}'}$$

$$U_{\mathrm{Dia}}(t) = \mathcal{P}\exp\left[i\int_{\vartheta_0}^{\vartheta} \sum_{n\neq m;p,q} F_{n,m}(\vartheta')G_{n,m}^{p,q}(\vartheta')d\vartheta' \right]$$

parameters $\mathbf{R}[\vartheta(t)]$
$|n_j^t\rangle \equiv |n_j^{\mathbf{R}}\rangle$: eigenbasis of $H(t)$
with energy $E_n(t)$ & degeneracy label j

degeneracy and crossings possible

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Gauge invariant formalism

$$U(t) \;=\; U_{\rm Dyn}(t)\; U_{\rm Geo}(t)\; U_{\rm Dia}(t)$$

$$U_{\rm Dia}(t) = \mathcal{P}\exp\left[i\int_{\vartheta_0}^{\vartheta}\sum_{n\neq m;p,q} F_{n,m}(\vartheta')G^{p,q}_{n,m}(\vartheta')d\vartheta'\right]$$

the geometric function

$$G^{p,q}_{n,m}(\vartheta) = U^{\dagger}_{\rm Geo}(\vartheta)|n^{\vartheta}_{p}\rangle\left(\langle n^{\vartheta}_{p}|i\frac{d}{d\vartheta}|m^{\vartheta}_{q}\rangle\right)\langle m^{\vartheta}_{q}|U_{\rm Geo}(\vartheta)$$

the modulation function

$$F_{n,m}(\vartheta) = e^{i\int_0^t[E_n(t')-E_m(t')]dt'}$$

$$H(\mathbf{R}(\vartheta)) = \textstyle\sum_{n,p} E_n(\vartheta)|n^{\vartheta}_{p}\rangle\langle n^{\vartheta}_{p}|$$

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nonadiabatic correction

$$U_{\mathrm{Dia}}(t) = \mathcal{P}\exp\left[i\int_{\vartheta_0}^{\vartheta}\sum_{n\neq m;p,q} F_{n,m}(\vartheta')G^{p,q}_{n,m}(\vartheta')d\vartheta'\right] \qquad F_{n,m}(\vartheta) = e^{i\int_0^t [E_n(t')-E_m(t')]dt'}$$

$$\sup_{\vartheta\in[\vartheta_0,\vartheta_T]}\left|\int_{\vartheta_0}^{\vartheta} F_{n,m}(\vartheta')d\vartheta'\right| = \xi_{\mathrm{avg}}, \qquad (n\neq m)$$

$$\boxed{\|U_{\mathrm{Dia}}(T) - I\| < \xi_{\mathrm{avg}}\left(g_{\mathrm{tot}}^2 + w_{\mathrm{tot}}\right)(\vartheta_T - \vartheta_0)}$$

where $g_{\mathrm{tot}} = \sum_{n\neq m} g_{n,m}$ and $w_{\mathrm{tot}} = \sum_{n\neq m} w_{n,m}$

$g_{n,m} \equiv \sup_{\vartheta\in[\vartheta_0,\vartheta_T]} ||\sum_{p,q} G^{p,q}_{n,m}(\vartheta)||$

$w_{n,m} \equiv \sup_{\vartheta\in[\vartheta_0,\vartheta_T]} ||\sum_{p,q} \frac{d}{d\vartheta} G^{p,q}_{n,m}(\vartheta)||$

unitarily invariant norm $||\cdot||$

A necessary and sufficient condition

$$\sup_{\vartheta\in[\vartheta_0,\vartheta_T]}\left|\int_{\vartheta_0}^{\vartheta} F_{n,m}(\vartheta')d\vartheta'\right| = \xi_{\text{avg}}, \qquad U_{\text{Dia}}(t) = \mathcal{P}\exp\left[i\int_{\vartheta_0}^{\vartheta}\sum_{n\neq m;p,q} F_{n,m}(\vartheta')G_{n,m}^{p,q}(\vartheta')d\vartheta'\right]$$

$$\boxed{\xi_{\text{avg}} \to 0}$$

sufficiency:

The condition $\xi_{\text{avg}} \to 0$ is sufficient because $F_{n,m}(\vartheta)$ are fast oscillating functions and the slowly varying functions $G_{n,m}^{p,q}(\vartheta)$ are averaged out. (Riemann-Lebesgue lemma)

necessity:

If the adiabatic limit $U_{\text{Dia}}(t) \to I$ is valid for arbitrary finite smooth paths, we can always find some paths which lead to $\xi_{\text{avg}} \to 0$.

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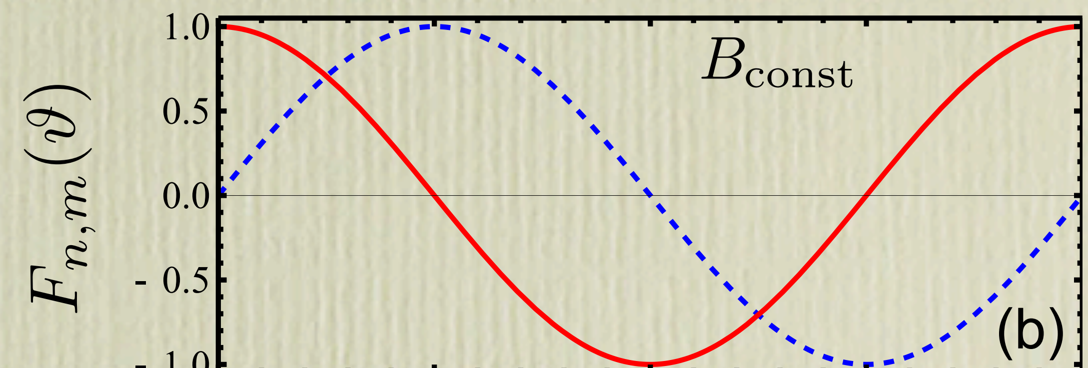
Adiabatic evolution

$$F_{n,m}(\vartheta) = e^{i\int_0^t [E_n(t') - E_m(t')]dt'}$$

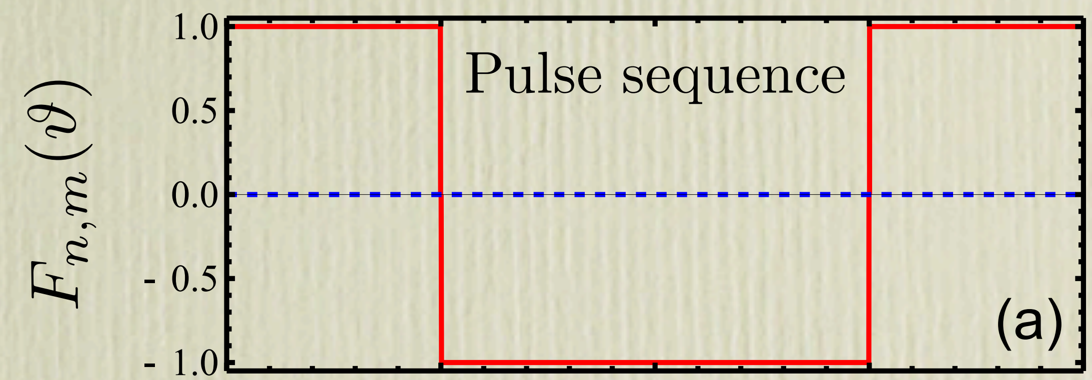
To have $$\sup_{\vartheta \in [\vartheta_0, \vartheta_T]} \left| \int_{\vartheta_0}^{\vartheta} F_{n,m}(\vartheta') d\vartheta' \right| \to 0$$

Large energy gaps
(i.e., slow parameter changes)



Pulse sequence
(similar to dynamical decoupling)

for DD, see: *Quantum Error Correction*,
Lidar and Brun, Cambridge University Press (2013)



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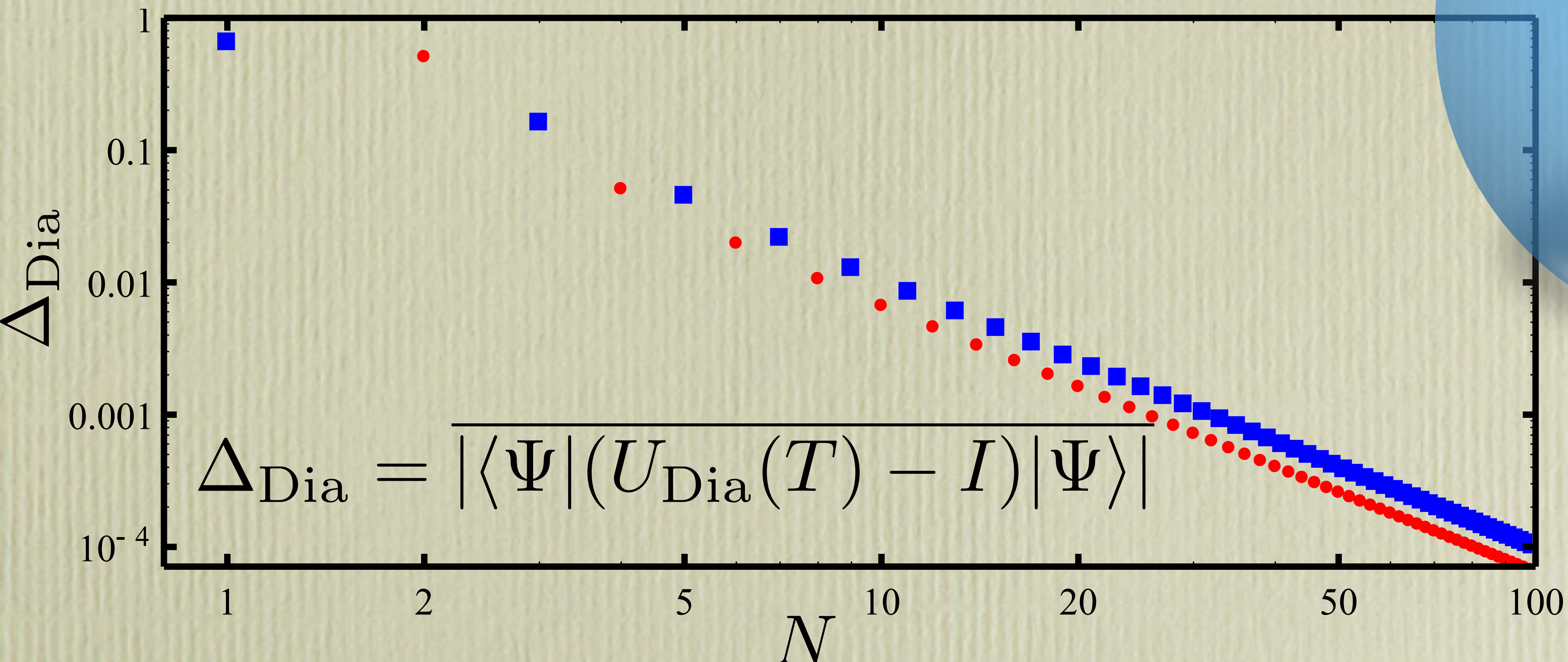
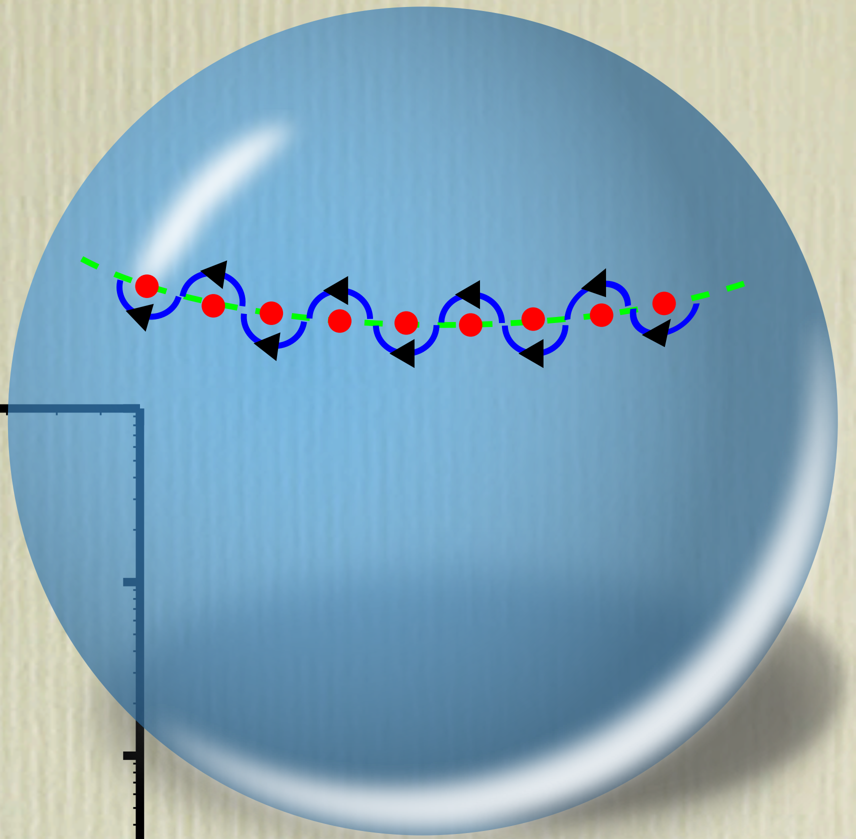
Adiabatic evolution by pulses





similar to eigenpath traversal by measurement,
evolution by phase randomization, but unitary

[Childs et al.] & [Sergio Boixo, E. H. Knill, Rolando Somma]

Fast continuous driving

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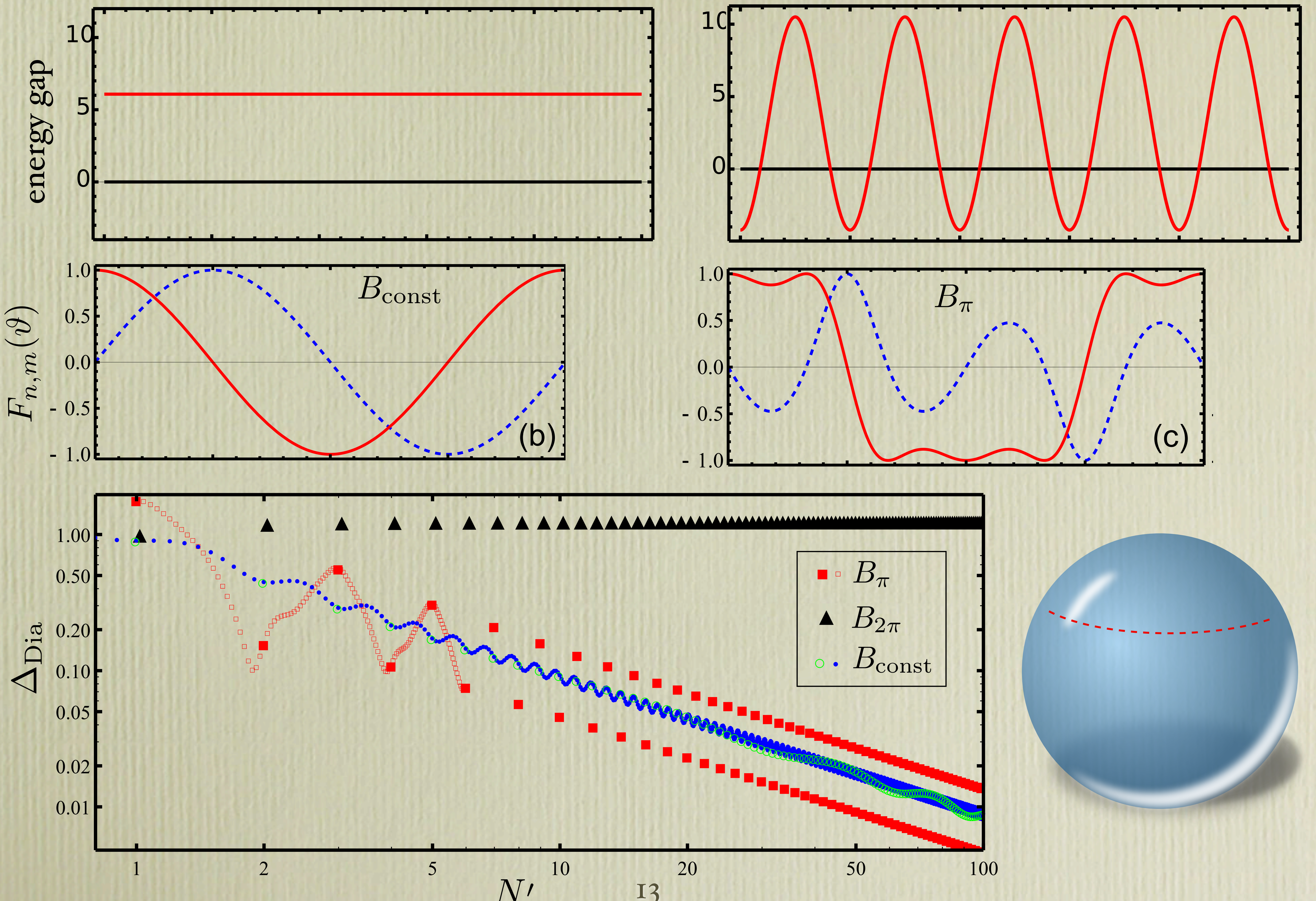

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Marzlin-Sanders inconsistency

(with possible degeneracy)

* A widely used quantitative condition:

$$\left|\frac{\langle n_p^t|\dot{m}_q^t\rangle}{E_n(t)-E_m(t)}\right| \ll 1$$

sufficient? No

necessary? No

If $U(t) = \mathcal{T}e^{-i\int_0^t H(t')dt'}$ is adiabatic, both $H(t)$ and $\bar{H}(t) = -U^\dagger(t)H(t)U(t)$ satisfy the condition.

But the evolution by $\bar{H}(t)$ is not adiabatic.

For the "bar" system:

$$|\bar{n}_j^t\rangle = U^\dagger(t)|n_j^t\rangle \qquad \bar{G}_{n,m}^{p,q}(\vartheta) = e^{-i\int_0^t [E_n - E_m]dt'} G_{n,m}^{p,q}(\vartheta)$$

$$\bar{U}_{\rm Dia}(t) = \mathcal{P}\exp\left[i\int_{\vartheta_0}^{\vartheta} \sum_{n\neq m;p,q} G_{n,m}^{p,q}(\vartheta')d\vartheta'\right]$$

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Collaboration with Martin B. Plenio

Part of the work was carried out at
The Chinese University of Hong Kong

香港中文大學
The Chinese University of Hong Kong

Acknowledge support from Ren-Bao Liu