Shortcuts to Adiabaticity: An overview

Adolfo del Campo

Theoretical Division
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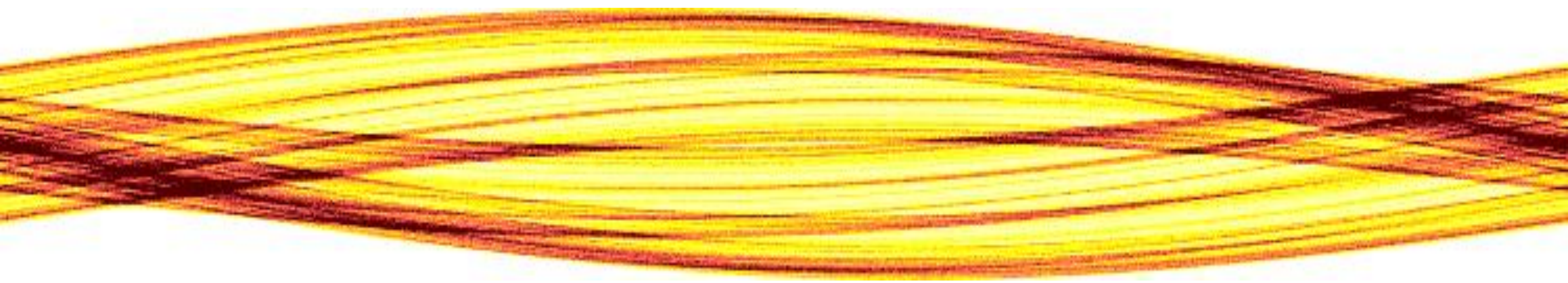

Telluride, CO, STA2014
July14th-18th 2014

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Conference Program

Exploring the interplay of

Shortcuts to Adiabaticity (STA)

with

Optimal Quantum Control

Finite-time Quantum Thermodynamics

Adiabatic Quantum Computation & Annealing

…

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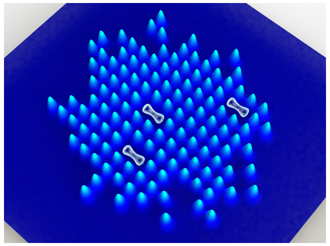
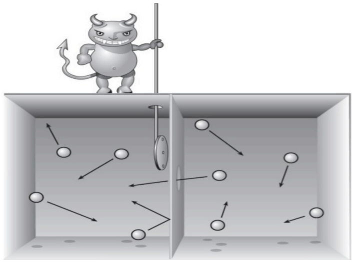
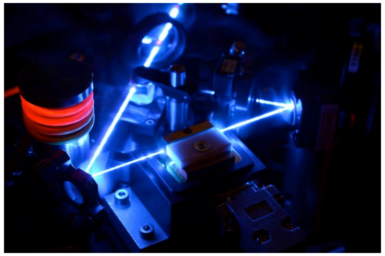
Shortcuts to adiabaticity: Why speeding things up?



Defect suppression in condensed matter systems and quantum simulation



Quantum thermodynamics
heat engines
ground state cooling



Quantum Information
Quantum annealing
Quantum Optics
Control of decoherence, noise and perturbations

Fast non-adiabatic processes to prepare a state mimicking adiabaticity

Review: Adv. At. Mol. Opt. Phys. **62,** 117 (2013)

Processes: Expansion, transport, splitting, adiabatic passage, phase transitions, …
Systems: ultracold atoms, ions chains, quantum dots, spin systems, NVC, …
Experiments: Nice, NIST, Mainz, PTB, MPQ, Florence, Trento, Tsukuba, …

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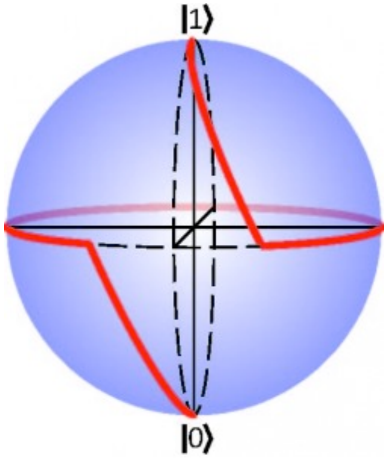
Contents

- PART I: Non critical systems
  - Inverting scaling laws
  - Counterdiabatic driving
  - Fast-forward technique
- PART II: Critical systems
  - Kibble-Zurek mechanism
  - Approaches to defect suppression
- Ultimate Quantum Speed Limits

PART I: STA
in noncritical systems



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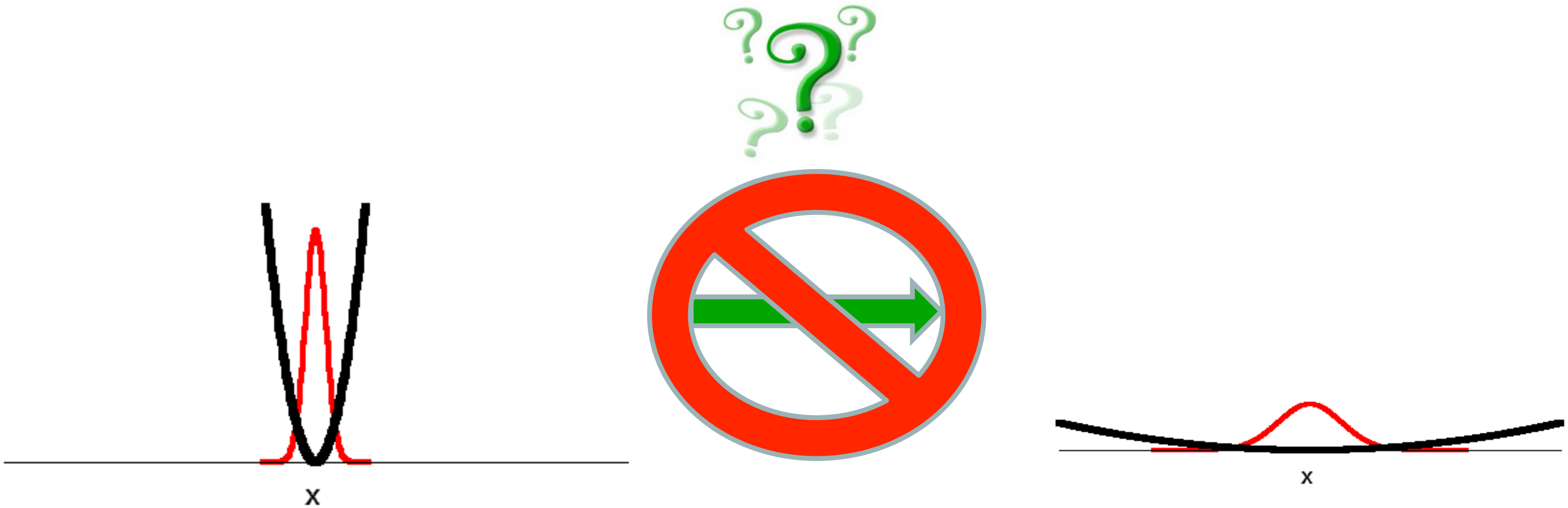
Inverting Scaling Laws

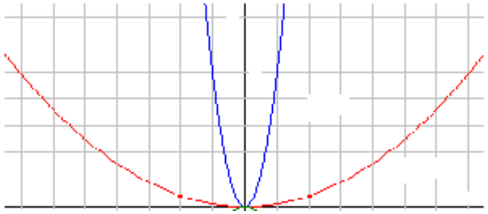
Inverting Scaling Laws



Standard expansion

Opening the trap

$$\omega(t) = \omega_i \left[1 + \frac{\omega_f - \omega_i}{\omega_i} \tanh \frac{t}{\tau} \right]$$



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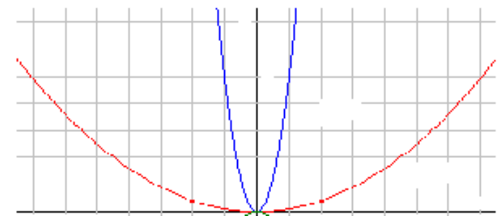
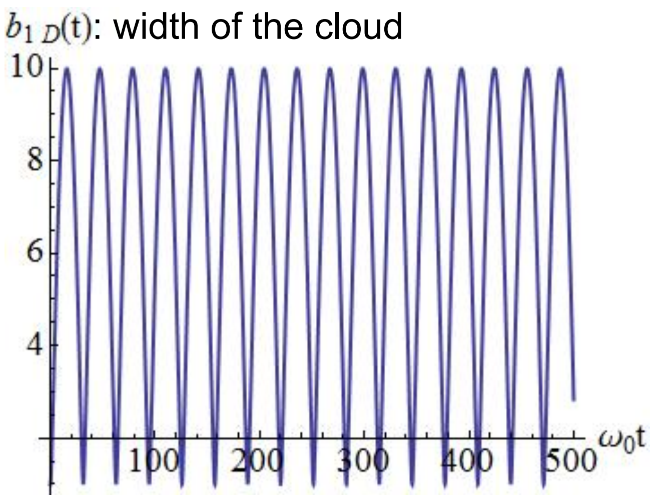
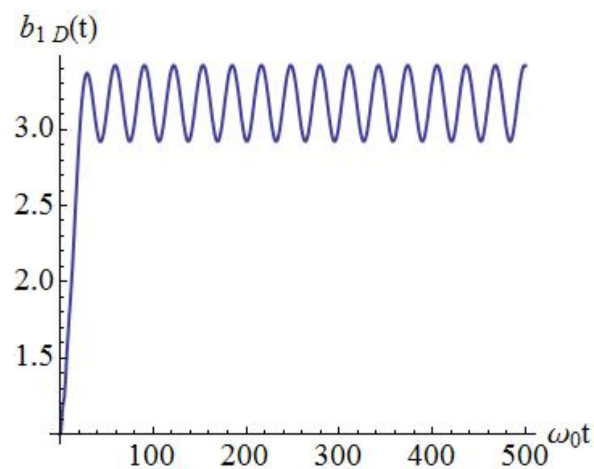
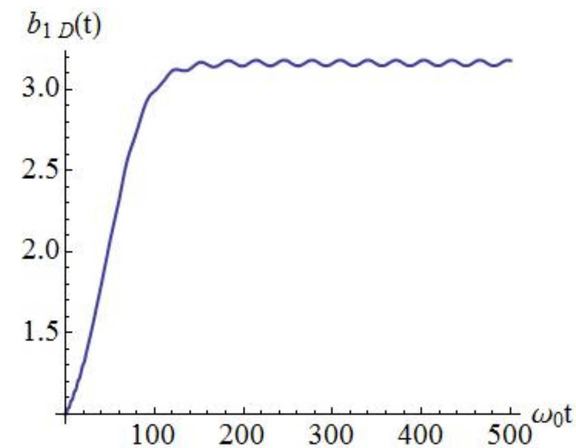


from sudden to adiabatic







Excitation of the breathing mode of the cloud

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Self-similar dynamics

1. Consider a time-dependent Hamiltonian harmonic oscillator

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega(t)^2x^2$$

$$\hat{H}\phi_n(x) = E_n\phi_n(x)$$

2. Impose a self-similar dynamical ansatz

$$\phi(x,t) = \frac{1}{b(t)^{1/2}}\exp\left[i\frac{m\dot{b}(t)}{2\hbar b(t)}x^2 - i\int_0^t \frac{E_n}{b(s)^2}ds\right]\phi\left[\frac{x}{b(t)}, t=0\right]$$

3. Get the consistency equation: scaling factor as function of trap frequency

$$\ddot{b} + \omega^2(t)b = \omega_0^2/b^3$$

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Lewis & Riesenfeld J. Math. Phys. **10**,1458 (1969)
Chen et al., Phys. Rev. Lett. **104**, 063002 (2010)

Self-similar dynamics

1. Take a somewhat general many-body time-dependent Hamiltonian

$$\hat{\mathcal{H}}=\sum_{i=1}^{N}\Big[-\frac{\hbar^2}{2m}\Delta_i^{(D)}+\frac{1}{2}m\omega^2(t)\mathbf{x}_i^2\Big]+\epsilon\sum_{i<j}\mathrm{V}(\mathbf{x}_{ij})\qquad \mathbf{x}_i\in\mathbb{R}^D,\ \mathbf{x}_{ij}=\mathbf{x}_i-\mathbf{x}_j$$

With a potential satisfying $\mathrm{V}(\lambda\mathbf{x})=\lambda^{\alpha}\mathrm{V}(\mathbf{x})$

2. Impose a self-similar dynamical ansatz

$$\Phi\left(\{\mathbf{x}_i\},t\right)=\frac{1}{b^{D/2}}e^{i\sum_{i=1}^{N}\frac{m\mathbf{x}_i^2\dot{b}}{2b\hbar}-i\mu\tau(t)/\hbar}\Phi\left(\{\frac{\mathbf{x}_i}{b}\},0\right)$$

3. Get the consistency equations, i.e.

$$\ddot{b}+\omega^2(t)b=\omega_0^2/b^3\qquad \epsilon(t)=b^{\alpha-2}$$

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del Campo, PRA **84**, 031606(R) (2011); PRL **111**, 100502 (2013)
Olshanii et al. Phys. Rev. Lett. **105**, 095302 (2010)

Design of a shortcut to adiabaticity

1. Force the scaling ansatz to reduce to the initial and final states considered

Boundary conditions:

$$b(0) = 1, \quad \dot{b}(0) = 0, \quad \ddot{b}(0) = 0$$
$$b(\tau) = \sqrt{\frac{\omega_f}{\omega_0}}, \quad \dot{b}(\tau) = 0, \quad \ddot{b}(\tau) = 0$$

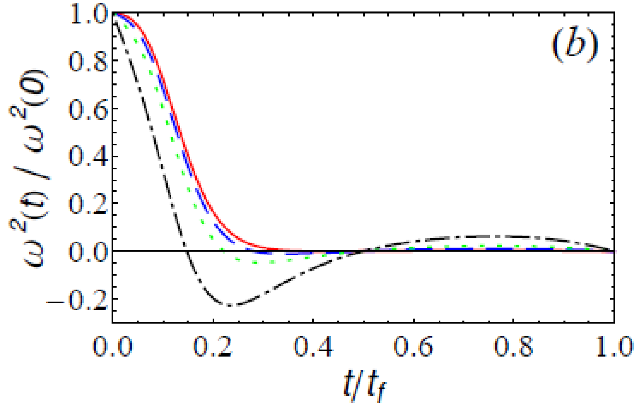
2. Determine an ansatz for the scaling factor (e.g. a polynomial)

$$b(t) = \sum_{j=0}^{5} a_j t^j$$

3. Find the driving frequency and coupling strength from the consistency equations

$$\ddot{b} + \omega^2(t) b = \omega_0^2 / b^3$$

$$\epsilon(t) = b^{\alpha-2}$$



Chen et al. Phys. Rev. Lett. **104**, 063002 (2010)
del Campo, PRA **84**, 031606(R) (2011)

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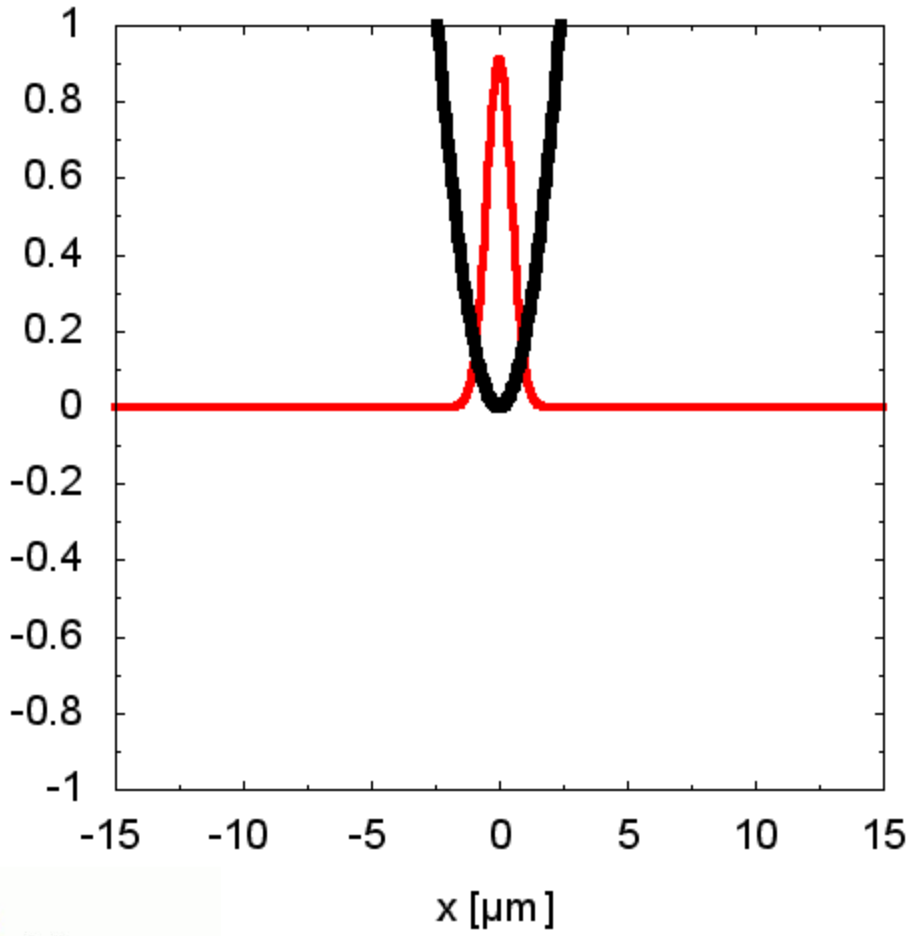
Example

Time Evolution:



$$\omega_0 = 250 \times 2\pi \; Hz$$

$$\omega_f = 2.5 \times 2\pi \; Hz$$

$$t_f = 2 \, ms$$

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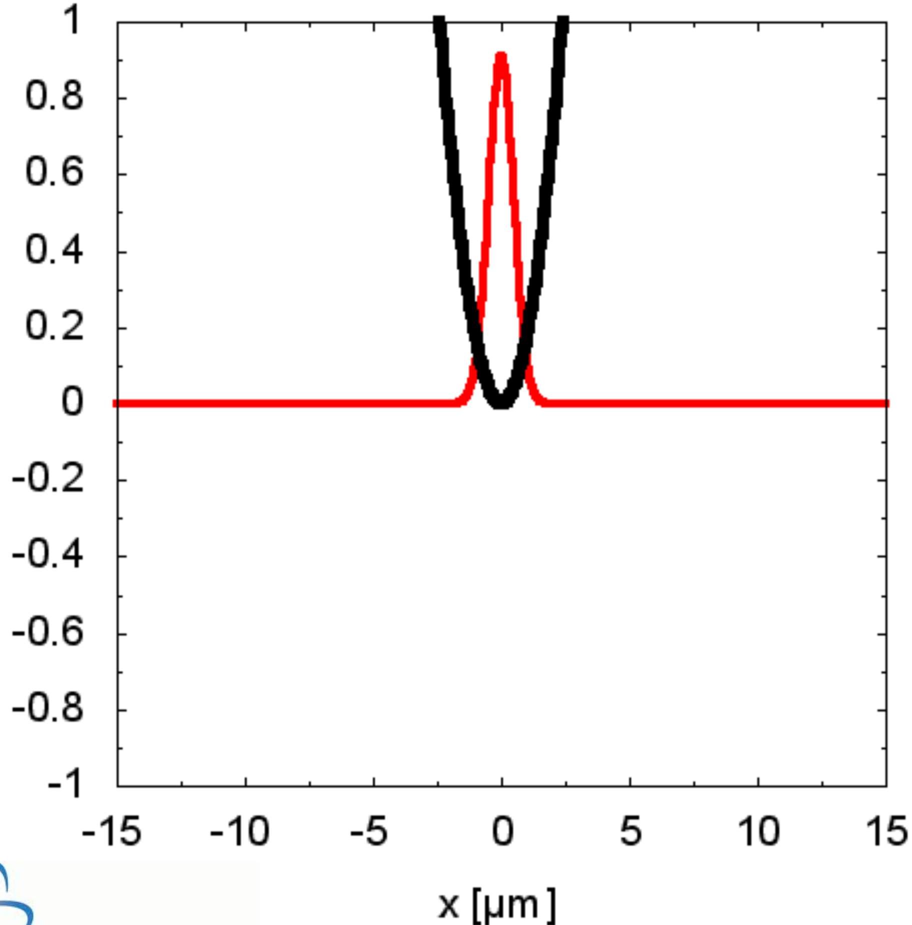
Example

Time Evolution:



$|\Psi(t,x)|^2$

$V(t,x)^2$

$\omega_0 = 250 \times 2\pi \; Hz$

$\omega_f = 2.5 \times 2\pi \; Hz$

$t_f = 2\, ms$

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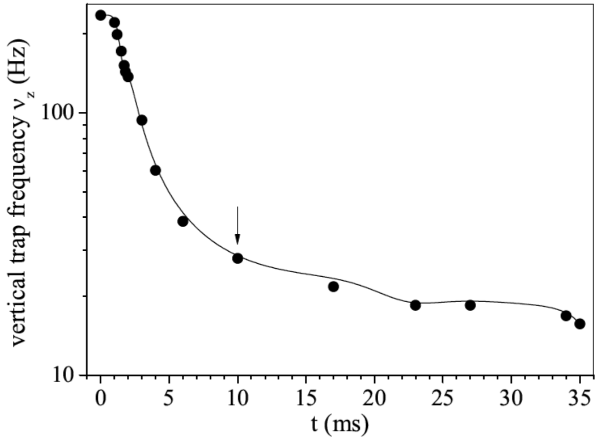
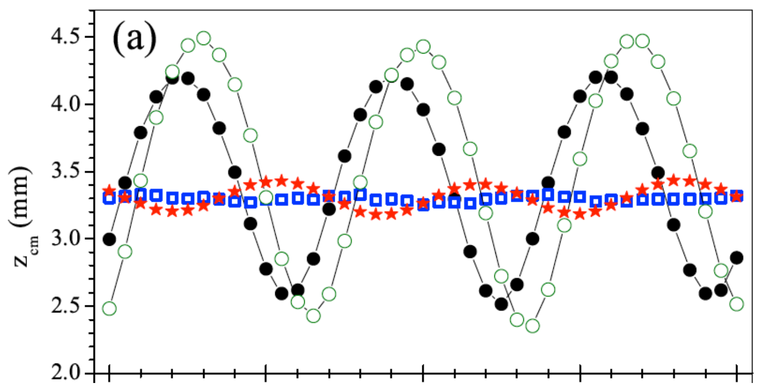
Experiments: expansion of a thermal cloud & BEC

Suppression of breathing mode excitation



Protocol: shortcuts to adibaticity



Linear vs shortcut BEC decompression

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Labeyrie's group @ Nice

Theory (single-particle)
Chen et al. Phys. Rev. Lett. **104**, 063002 (2010)
Experiments (single-particle / mean-field BEC)
J.-F. Schaff et al. EPL **93**, 23001 (2011)
J.-F. Schaff et al. Phys. Rev. A **82**, 033430 (2010)

Inverting Scaling Laws: applications

Phase-space preserving ground-state cooling (state mapping)

Chen et al 10 (SHO)

Salamon-Hoffmann-Rezek-Kosloff 11 (SHO)

Boshier-AdC 12 (box)

Reformulating the third law of thermodynamics (Kosloff's talk)

Superadiabatic classical and quantum engines

Working medium: TD SHO

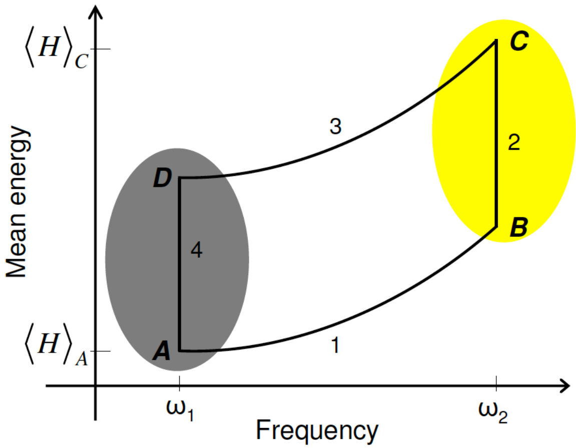
Schmiedl-Siefert 07 (underdamped Brownian & quantum)

Salamon-Hoffmann-Rezek-Kosloff 09 (OQC)

AdC-Goold-Paternsotro 14 (quantum)

Deng et al13 (classical & quantum)

Zu 14 (brownian working medium)



Counterdiabatic driving

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Counterdiabatic driving

Consider driving a system Hamiltonian

$$\hat{H}_0(t)|n(t)\rangle = E_n(t)|n(t)\rangle$$

Write the adiabatic approximation

$$|\psi_n(t)\rangle = \exp\left[-\frac{i}{\hbar}\int_0^t E_n(s)ds - \int_0^t \langle n(s)|\partial_s n(s)\rangle ds\right]|n(t)\rangle$$

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Counterdiabatic driving

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Is there a Hamiltonian for which the adiabatic approximation is exact?

$$i\hbar\partial_t|\psi_n(t)\rangle = \hat{H}(t)|\psi_n(t)\rangle$$

Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_n \left(|\partial_t n\rangle\langle n| - \langle n|\partial_t n\rangle|n\rangle\langle n|\right)$$

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Counterdiabatic driving

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Yes, indeed!

$$\hat{H}(t) \equiv \hat{H}_0(t) + \hat{H}_1(t)$$

$$\hat{H}_1(t) = i\hbar\sum_{n\neq m}\sum_m \frac{|m\rangle\langle m|\partial_t \hat{H}_0|n\rangle\langle n|}{E_n(t)-E_m(t)}$$

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Theory: Demirplak & Rice 2003; = M. V. Berry 2009 “Transitionless quantum driving”
Experiment for TLS: Morsch’s group Nature Phys. 2012; NVC: Suter’s group PRL 2013

Counterdiabatic driving: applications

Counterdiabatic terms are often **nonlocal**

Search for experimentally-realizable local Unitarily equivalent Hamiltonians (e.g. Deffner's talk)

$$\hat{H}' = U\hat{H}U^\dagger - i\hbar U\partial_t U^\dagger$$

RAP in Two level system (spin flip) $\hat{H}_1 \propto \sigma_y$ $\hat{H}'_1 \propto \sigma_z$

Demirplak & Rice 2003 Bason et al 2012

Time-dependent harmonic oscillator $\hat{H}_1 \propto (xp + px)$ $\hat{H}'_1 \propto x^2$

Muga el at 2010, Jarzynski 2013 Ibáñez et al 12, AdC 13

Transport of matter waves $\hat{H}_1 \propto p$ $\hat{H}'_1 \propto x$

Deffner-Jarzynski-AdC 14

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Theory: Demirplak & Rice 2003; M. V. Berry 2009
Experiment for TLS: Morsch's group Nature Phys. 2012; NVC: Suter's group PRL 2013

Many-body systems?

With dynamical symmetries (e.g. self-similarity) required driving is almost as in the single-particle case

Quantum fluids: scaling laws & counterdiabatic driving

Family of interacting quantum fluids

$$\hat{H}_0(t) = \sum_{i=1}^{N}\left[-\frac{\hbar^2}{2m}\Delta_{\mathbf{q}_i} + \frac{1}{2}m\omega^2(t)\mathbf{q}_i^2 + U(\mathbf{q}_i,t)\right] + \epsilon(t)\sum_{i<j}V(\mathbf{q}_i-\mathbf{q}_j)$$

$$U(\mathbf{q},t) = \frac{1}{\gamma^2(t)}U\left(\frac{\mathbf{q}}{\gamma(t)},0\right), \qquad \mathrm{V}(\lambda\mathbf{q}) = \lambda^{-\alpha}\mathrm{V}(\mathbf{q})$$

Spectral properties generally unavailable

Scaling ansatz $$\Phi(t) = \gamma^{-\frac{ND}{2}}e^{-i\mu\tau(t)/\hbar}\Phi\left[\frac{\mathbf{q}_1}{\gamma(t)},\dots,\frac{\mathbf{q}_N}{\gamma(t)};0\right]$$

Nonlocal auxiliary Hamiltonian $$\hat{H}_1 = -i\frac{\hbar\dot{\gamma}}{2\gamma}\sum_{i=1}^{N}(\mathbf{q}_i\partial_{\mathbf{q}_i} + \partial_{\mathbf{q}_i}\mathbf{q}_i)$$

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A. del Campo, PRL 111, 100502 (2013)

Quantum fluids: scaling laws & counterdiabatic driving

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Spectral properties generally unavailable

Scaling ansatz $\Phi(t) = \gamma^{-\frac{ND}{2}}e^{-i\mu\tau(t)/\hbar}\Phi\left[\frac{\mathbf{q}_1}{\gamma(t)},\dots,\frac{\mathbf{q}_N}{\gamma(t)};0\right]$

Allowing excitations $\mathcal{U} = \prod_{i=1}^{N}\exp\left(\frac{im\dot{\gamma}}{2\hbar\gamma}\mathbf{q}_i^2\right), \Phi(t)\rightarrow\Psi(t) = \mathcal{U}\Phi(t)$

Local driving $\hat{\mathcal{H}}_1 = -\frac{1}{2}m\frac{\ddot{\gamma}}{\gamma}\sum_{i=1}^{N}\mathbf{q}_i^2$

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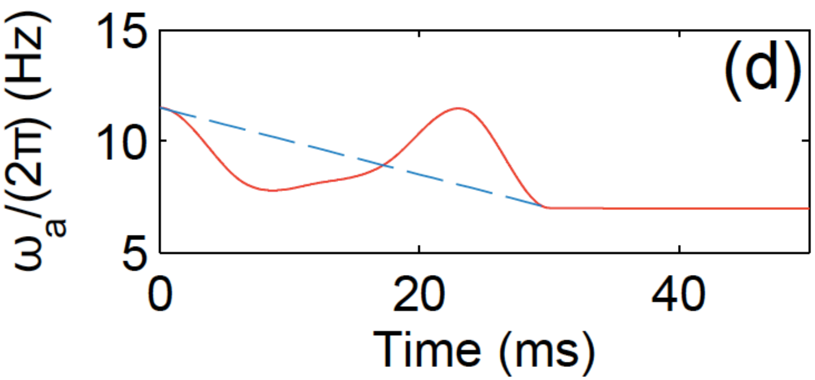
A. del Campo, PRL 111, 100502 (2013)

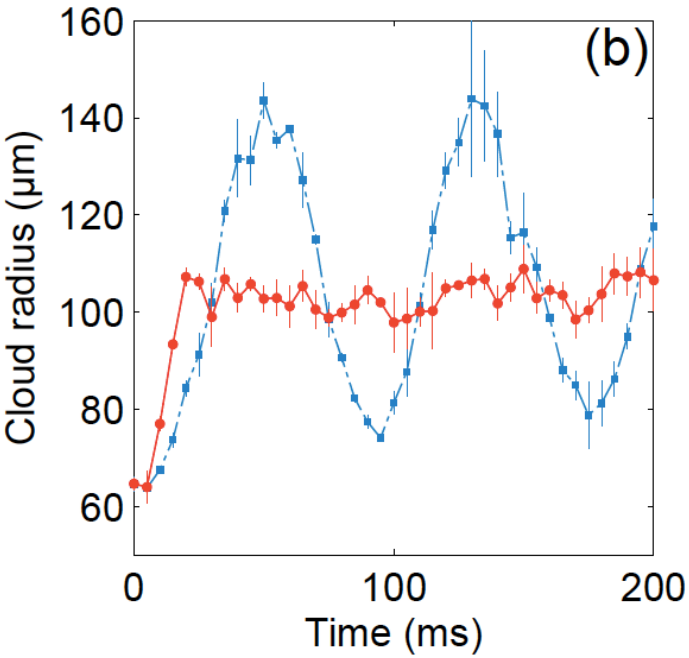
Experiment: many-body shortcuts

Scaling of phonons and shortcuts to adiabaticity in a one-dimensional quantum system

W. Rohringer,[1] D. Fischer,[1] F. Steiner,[1] I. E. Mazets,[1,2] J. Schmiedmayer,[1] and M. Trupke[1]
[1] *Vienna Center for Quantum Science and Technology, Atominstitut, TU Wien, 1020 Vienna, Austria*
[2] *Ioffe Physical-Technical Institute of the Russian Academy of Sciences, 194021 St. Petersburg, Russia*
(Dated: December 23, 2013)

Shortcut vs standard expansion





Fast-forward technique

Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

Fast-forward technique

Consider the dynamics (mean-field)

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + (\mathcal{V}+\mathcal{V}_{\rm au})\Psi + g|\Psi|^2\Psi,$$

Ansatz for the evolution

$$\Psi(\mathbf{q},t) = \psi[\mathbf{q},R(t)]e^{i\phi(\mathbf{q},t)}e^{-\frac{i}{\hbar}\int_0^t \mu[R(t')]dt'}$$

where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

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Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

Fast-forward technique

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where

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \mathcal{V}\psi + g|\psi|^2\psi = \mu\psi.$$

Substituting ansatz, separating real and imaginary parts

$$\mathcal{V}_{\rm au}({\rm q},t) = -\frac{\hbar^2}{2m}(\nabla\phi)^2 - \hbar\partial_t\phi$$

$$\nabla^2\phi + 2\nabla\ln\psi\cdot\nabla\phi + \frac{2m}{\hbar}\dot{R}\partial_R\ln\psi = 0$$

determine the auxiliary driving potential

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Theory: Masuda & Nakamura 2008, 2010, 2011
Experiments: ???

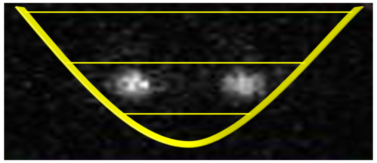
Fast-forward technique: application

For self-similar processes is equivalent to other techniques

Example: transport of ion chains/strongly correlated systems (beyond mean-field)

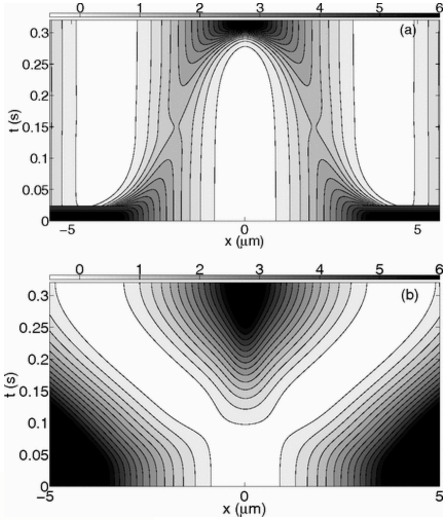
(Masuda PRA 2012)

Auxiliary potential = linear potential

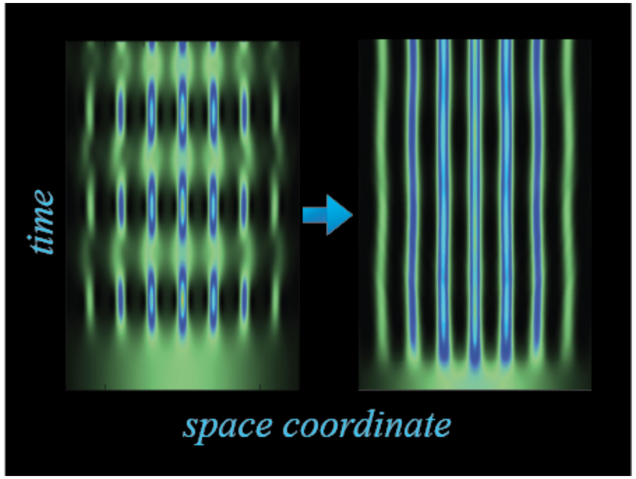


“Favourite” technique for non-self similar driving of matter-waves

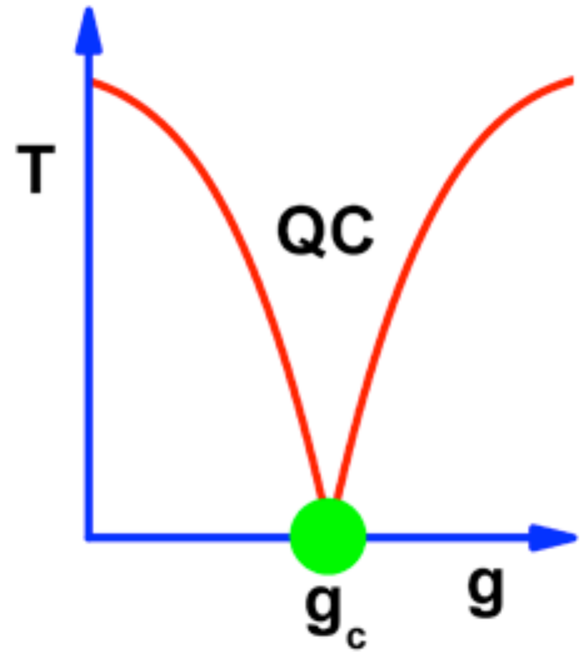
Matter wave splitting
(Torrontegui et al PRA 2013)



Loading an optical lattice
(Masuda, Nakamura, AdC PRL 2014)
Auxliary potential ≈ bichromatic lattice

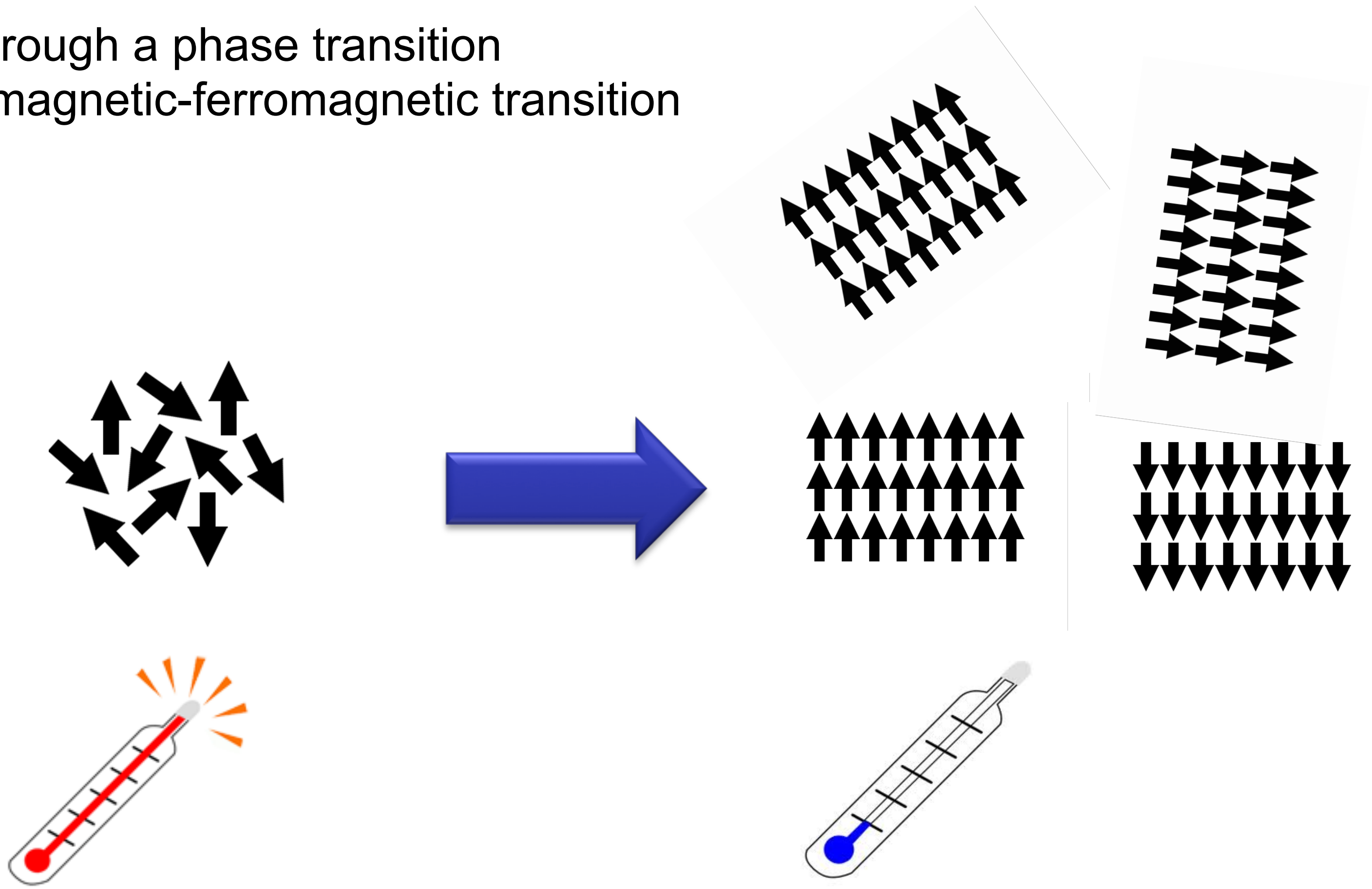


PART II:
STA
in critical systems



Spontaneous symmetry breaking

Driving through a phase transition
e.g. paramagnetic-ferromagnetic transition

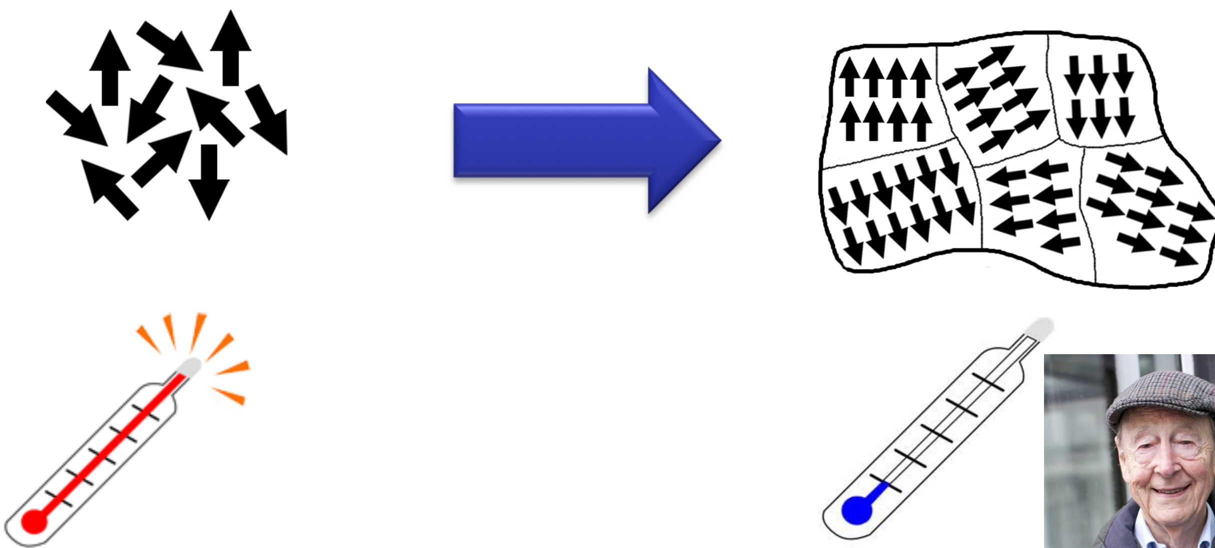


Various ground states with same energy
(ground state manifold)

Spontaneous symmetry breaking

Driving through a phase transition
e.g. paramagnetic-ferromagnetic transition

Cooling at finite rate!



Broken symmetry
how big are the pieces?
how many defects?
The Kibble-Zurek mechanism

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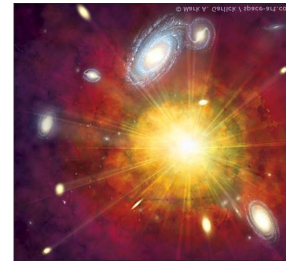
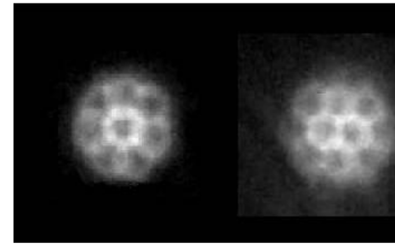
Cosmology in the lab




- Cosmology : symmetry breaking during expansion and cooling of the early universe

- Condensed matter:
  - Vortices in Helium
  - Liquid crystals
  - Superconductors
  - Superfluids




Landau theory: Similar free-energy landscape near a critical point

Kibble-Zurek mechanism: formation of defects

T. W. B. Kibble, JPA 9, 1387 (1976); Phys. Rep. 67, 183 (1980)
W. H. Zurek, Nature (London) 317, 505 (1985); Acta Phys. Pol. B. 1301 (1993)

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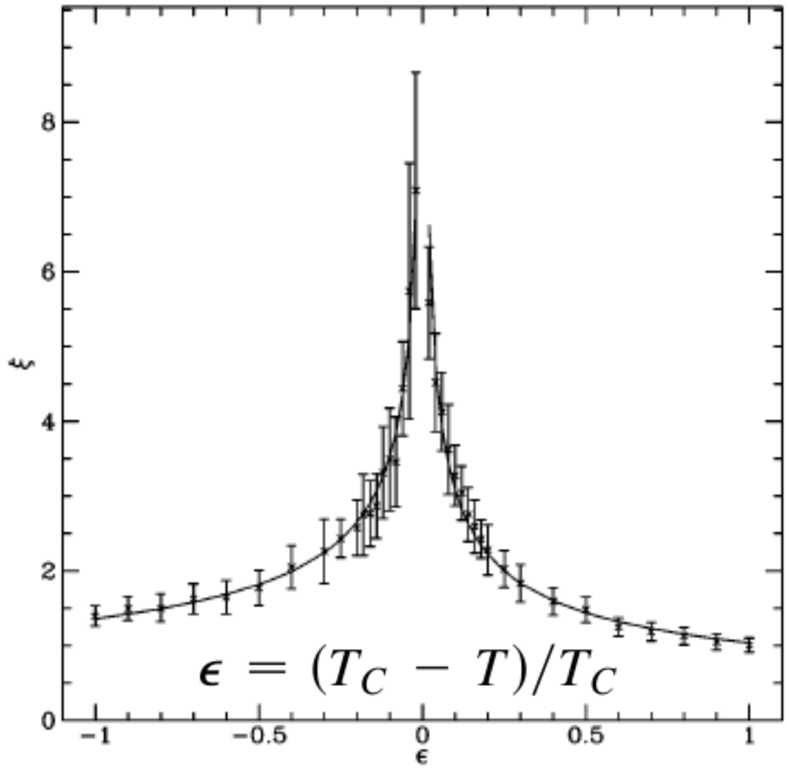
Second order phase transitions

Universal behaviour of the order parameter: divergence of

Correlation/healing length $$\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^{\nu}}$$

Dynamical relaxation time $$\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$$



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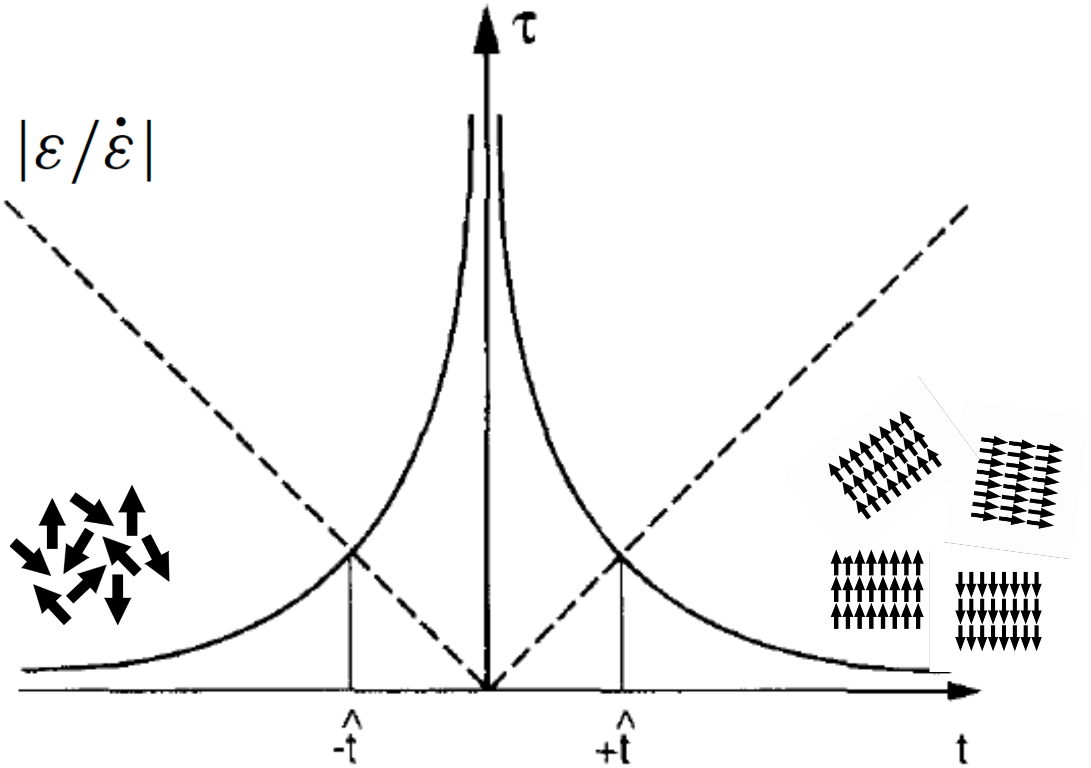
The Kibble-Zurek mechanism



Linear quench

$$\varepsilon(t) = t/\tau_Q$$

$$\tau(t) = \frac{\tau_0}{|\varepsilon(t)|^{z\nu}}$$

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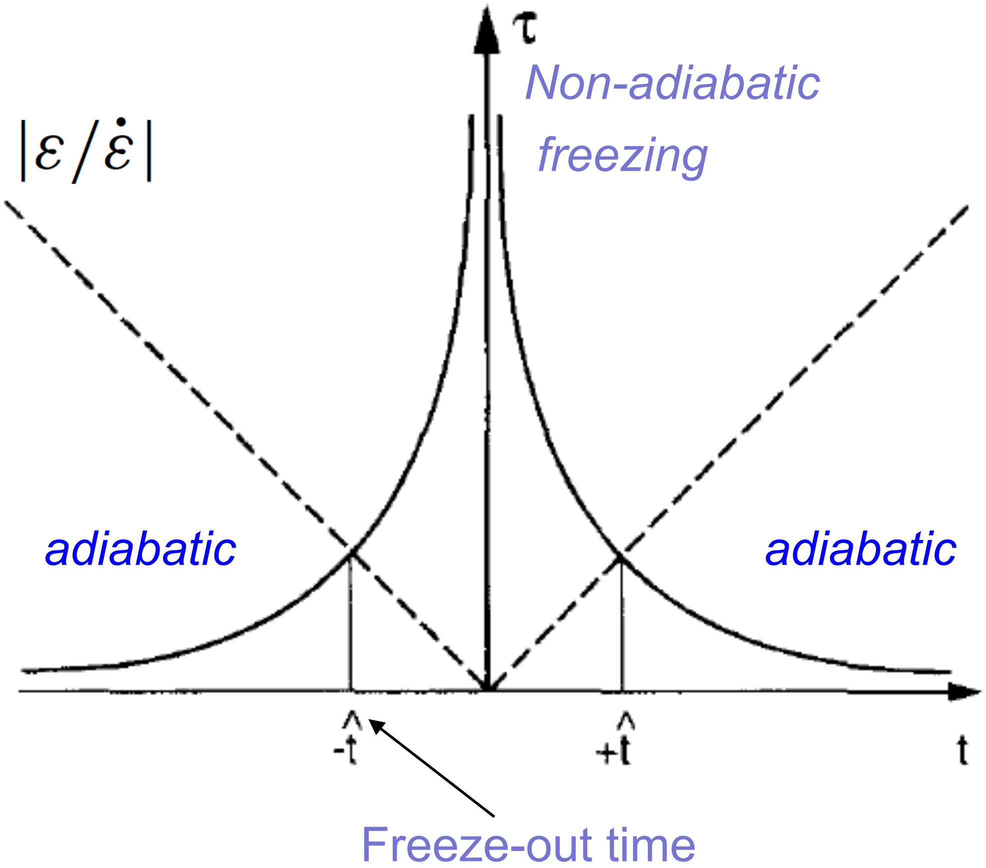

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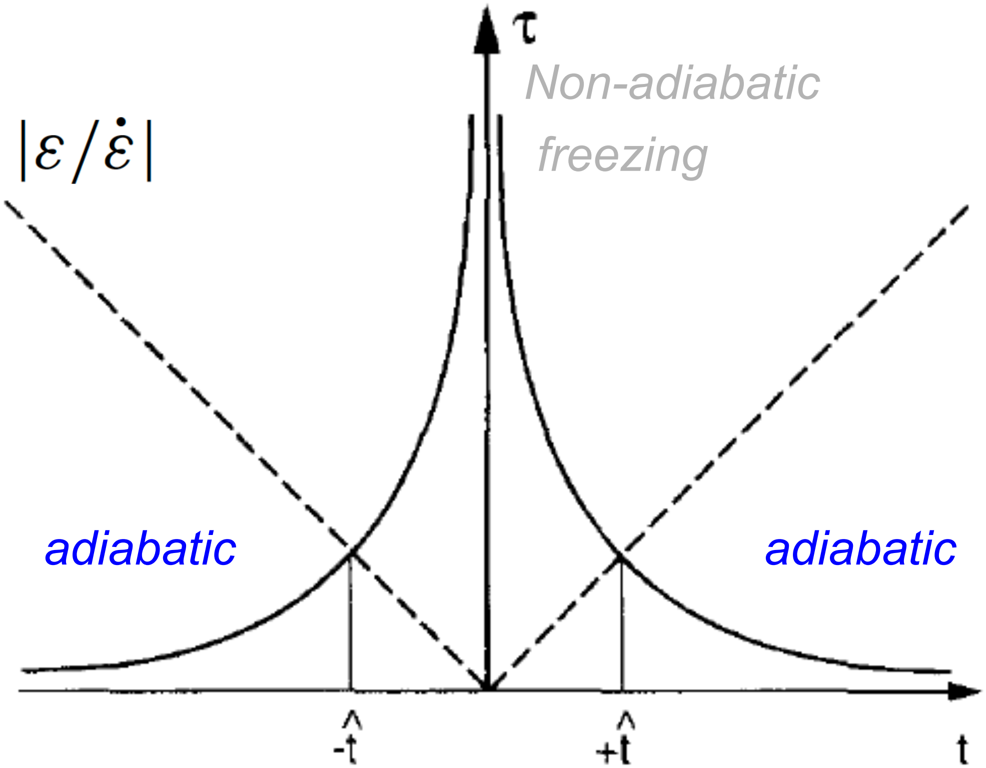

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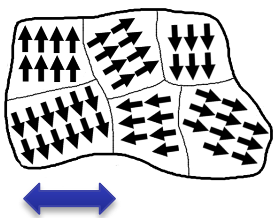

Average domain size given by the equilibrium correlation length at the freeze-out time

$$\xi(t) = \frac{\xi_0}{|\varepsilon(t)|^{\nu}} \qquad \hat{\xi} = \xi(\hat{t}) = \xi_0 \left(\frac{\tau_Q}{\tau_0}\right)^{\frac{\nu}{1+z\nu}}$$

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Ways out of the Kibble-Zurek mechanism?

How to suppress defect formation?

Some approaches include:

- Finite-system size (Murg-Cirac 04)
- Nonlinear power-law quenches (Polkovnikov & Barankov 08, Sen-Sengupta-Mondal 08)
- Dissipation (Patane et al 08)
- Inhomogeneous driving (Kibble-Volovik 97, Zurek 09, Dziarmaga-Rams 10, AdC et al 10)
- Optimal quantum control (Doria-Calarco-Montangero 11, Caneva-Calarco-Fazio-Santoro-Montangero 11)
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J. Phys.: Condens. Matter **25** (2013) 404210 (10pp) doi:10.1088/0953-8984/25/40/404210

Causality and non-equilibrium second-order phase transitions in inhomogeneous systems

A del Campo[1,2], T W B Kibble[3] and W H Zurek[1]

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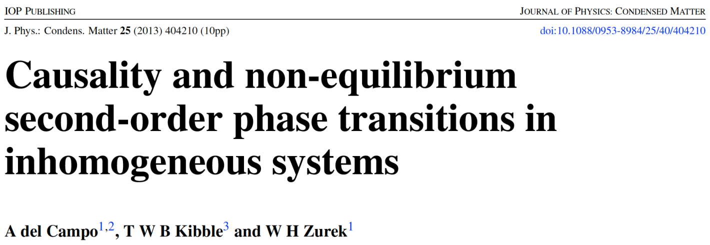
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J. Phys.: Condens. Matter **25** (2013) 404210 (10pp) — doi:10.1088/0953-8984/25/40/404210

Causality and non-equilibrium second-order phase transitions in inhomogeneous systems

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Structural phases in trapped ions

N ions on a ring trap with harmonic transverse confinement

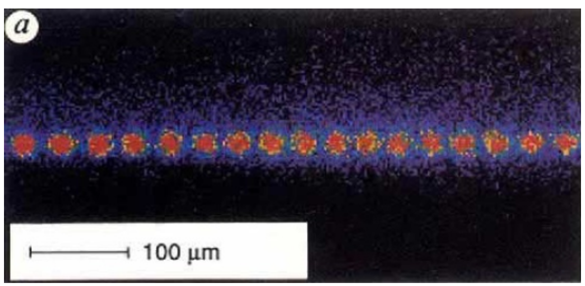
$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$

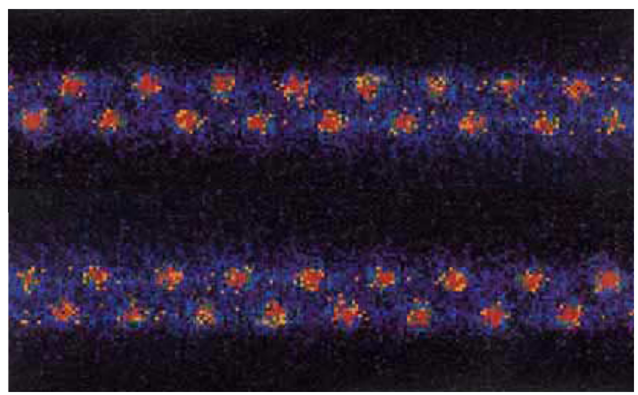
Critical transverse frequency

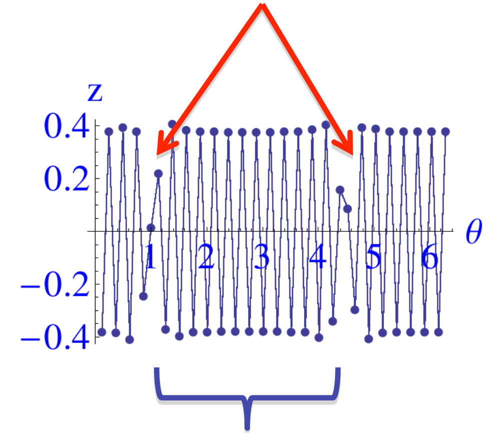
Linear chain

Degenerated zig-zag chains

Excitations (kinks)







$$\nu_t^{(c)2} = 4\frac{Q^2}{ma(0)^3}$$

Domain of size

$$\hat{\xi}_x = a\omega_0(\tau_Q/\delta_0)^{1/3}$$

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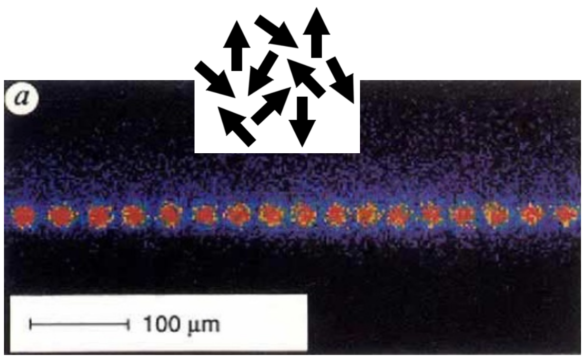

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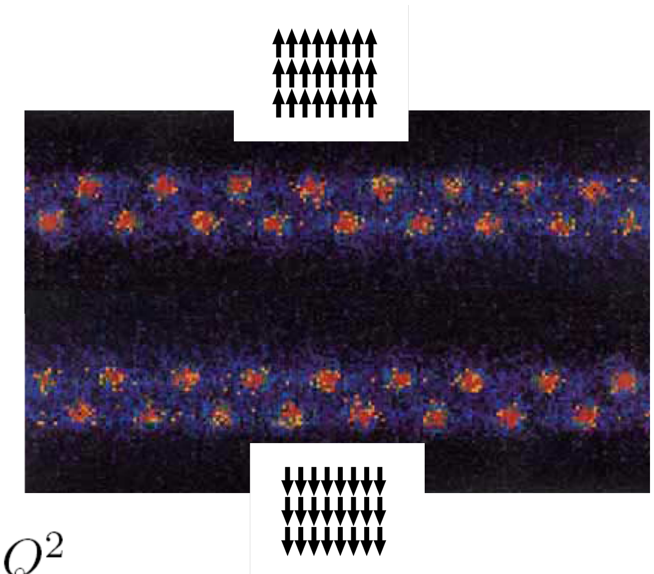

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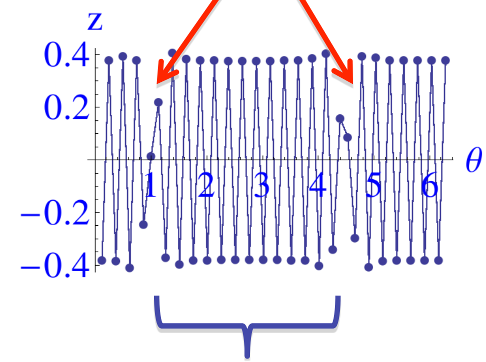

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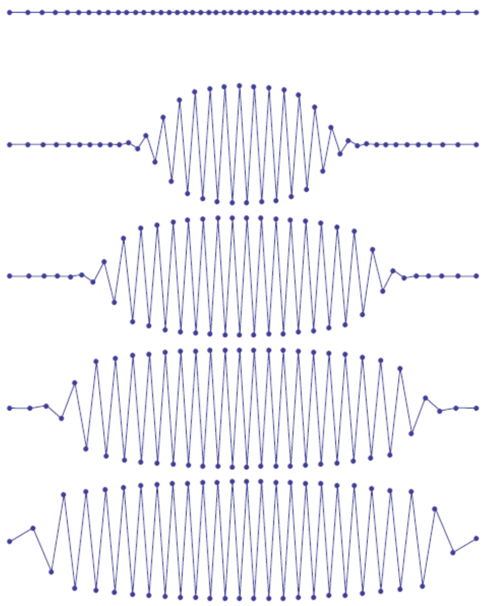
Inhomogeneous driving

Axial and transverse harmonic potential (instead of a ring trap)

$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2 + \nu^2 x_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$



$$\nu_t^{(c)2} = 4\frac{Q^2}{ma(0)^3}$$

$$\nu_t^{(c)}(x)^2 = 4\frac{Q^2}{ma(x)^3}$$

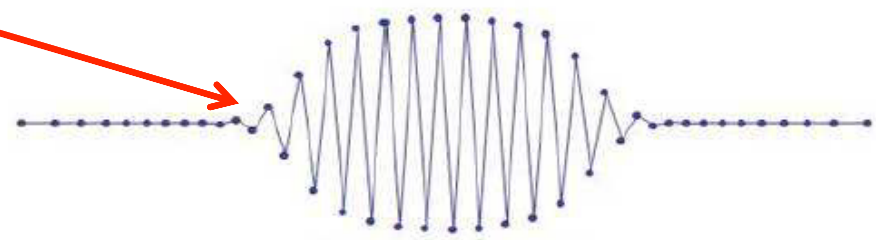
Spatially dependent critical frequency (within LDA)

Inhomogeneous driving

Causality restricts the effective size of the chain

Front velocity v_F

Sound velocity c



Adiabatic dynamics $v_F < c$

Kink formation $v_F > c$ in an effective system size $L_{\rm eff}(\tau_Q)$

Density of defects: New power law $$d = \frac{L_{\rm eff}(\tau_Q)}{\hat{\xi}}\frac{1}{L} \sim \left(\frac{1}{\tau_Q}\right)^{4/3}$$

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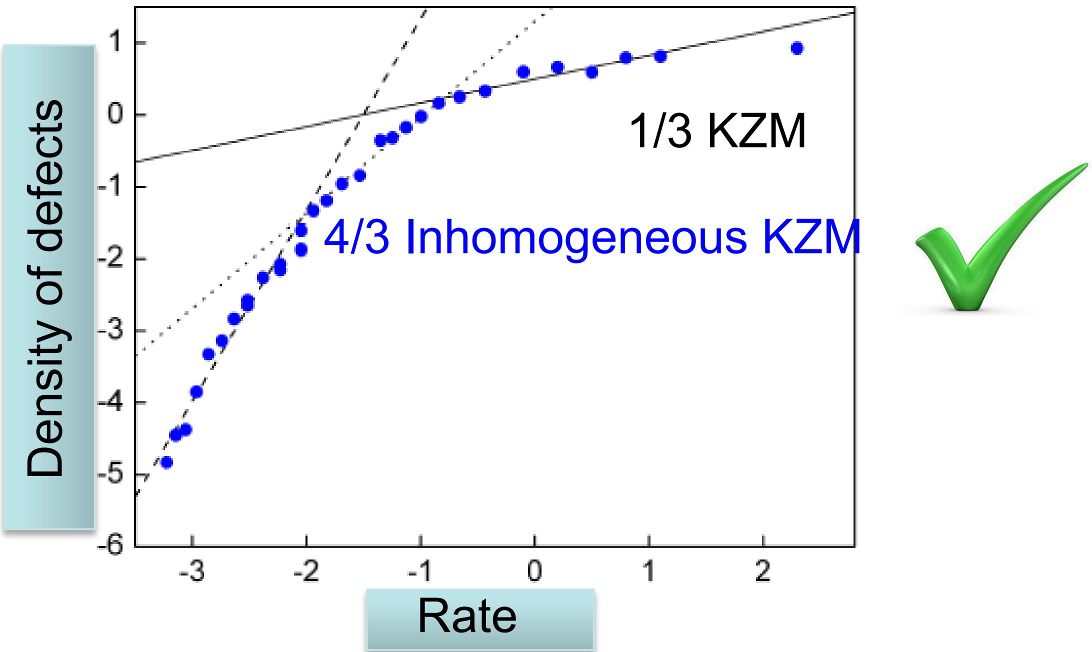
AdC et al. *PRL* 105, 075701 (2010)

Testing KZM in the lab

Axial and transverse harmonic potential (instead of a ring trap)

$$H = \frac{1}{2}m\sum_n \dot{\boldsymbol{r}}_n^2 + \frac{1}{2}m\sum_n(\nu_t^2 z_n^2 + \nu^2 x_n^2) + \frac{Q^2}{2}\sum_{n\neq n'}\frac{1}{|\boldsymbol{r}_n - \boldsymbol{r}'_n|}$$



MD numerics: Langevin dynamics including laser cooling (damping)
N=50, 2000 realizations, quench of the transverse trapping frequency

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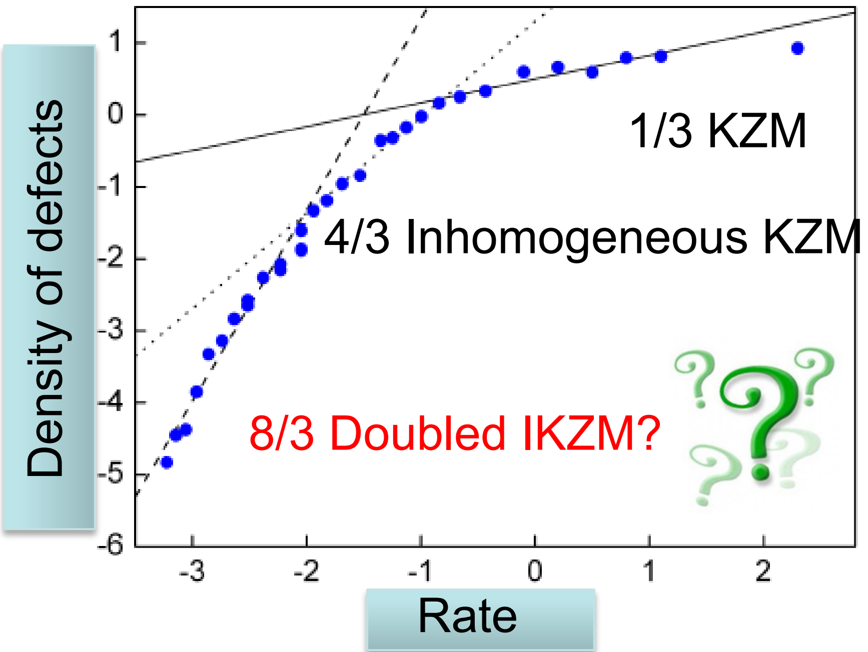

With only {0,1} defects
third scaling:
Rivers et al. Doubling of IKZM?

$$p_1 \simeq \left[\frac{2\hat{x}}{\hat{\xi}(0)}\right]^2 \propto \left(\frac{\tau_0}{\tau_Q}\right)^{\frac{2(1+2\nu)}{1+\nu z}} = \left(\frac{\tau_0}{\tau_Q}\right)^{\frac{8}{3}}$$

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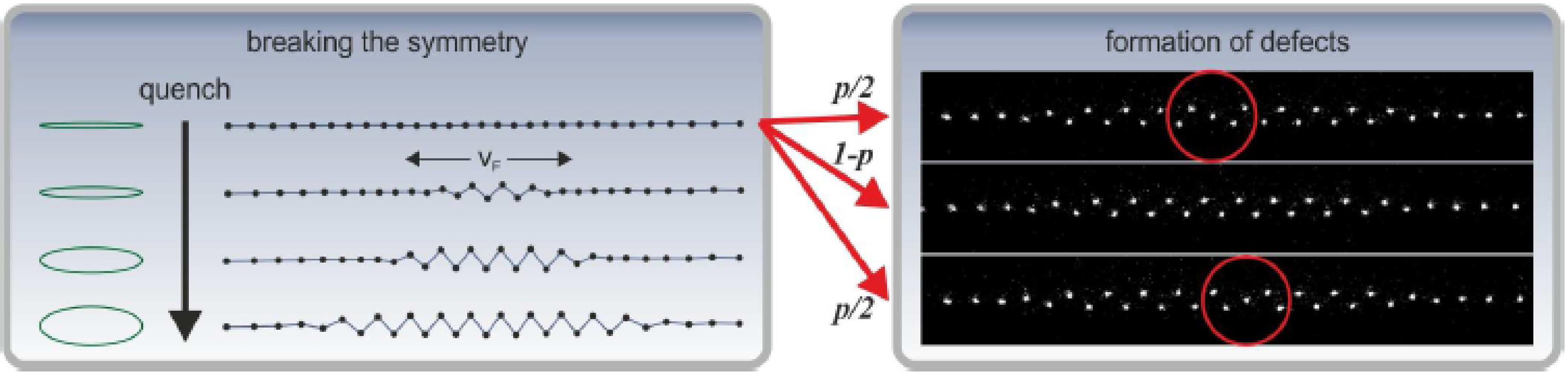
First Experiment - Collaboration with T. E. Mehlstaubler's group at PTB



32 ions, only {0,1} defects per realization

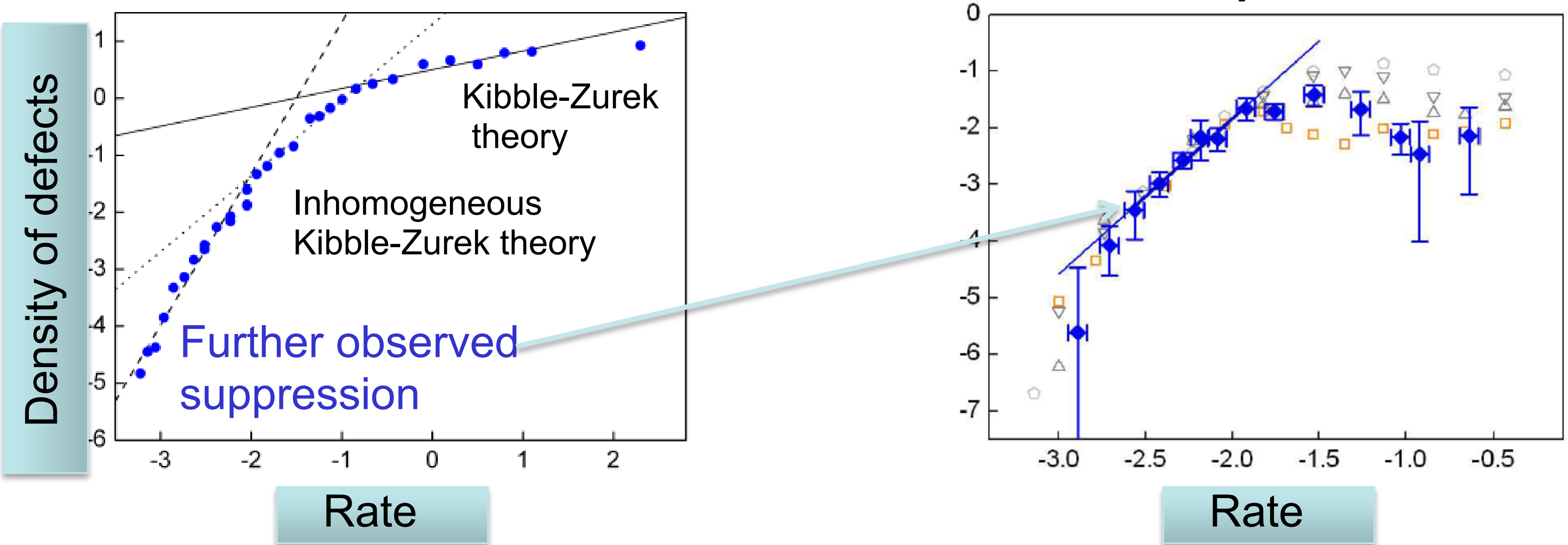
Experiments & numerics

Collaboration with T. E. Mehlstaubler's group at PTB

Kibble-Zurek theory fails at the onset of adiabatic dynamics (few excitations)

Regime of interest to quantum simulation

Numerics **Experiment**



Received 25 Mar 2013 | Accepted 11 Jul 2013 | Published 7 Aug 2013
DOI: 10.1038/ncomms3291

Topological defect formation and spontaneous symmetry breaking in ion Coulomb crystals

K. Pyka[1,*], J. Keller[1,*], H.L. Partner[1], R. Nigmatullin[2,3], T. Burgermeister[1], D.M. Meier[1], K. Kuhlmann[1], A. Retzker[4], M.B. Plenio[2,3,5], W.H. Zurek[6], A. del Campo[6,7] & T.E. Mehlstäubler[1]

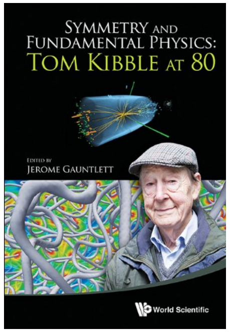
32 ions, only {0,1} defects per realization
Pyka et al. Nature Communications 4, 2291 (2013)

Comparison

$n \propto \tau_Q^{-\alpha}$.

Group	Number of ions	Kink number	Fitted exponent α
Mainz University[14]	16	$\{0, 1\}$	2.68 ± 0.06
PTB[15]	29 ± 2	$\{0, 1\}$	2.7 ± 0.3
Simon Fraser University[13]	42 ± 1	$\{0, 2\}$	3.3 ± 0.2

A. del Campo, W. H. Zurek
Int. J. Mod. Phys. A 29, 1430018 (2014)



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Inhomogeneous driving

Experimental tests restricted to **small systems & low number of defects**

Onset of adiabatic dynamics lacks theory

Inhomogeneous driving enhances role of causality

Partial applicability to **adiabatic quantum computation**

It does NOT require diagonalization of the Hamiltonian

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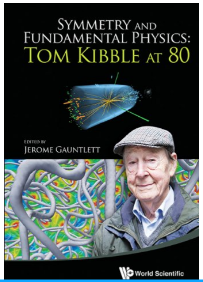
Causality and non-equilibrium second-order phase transitions in inhomogeneous systems

A del Campo[1,2], T W B Kibble[3] and W H Zurek[1]

[1] Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
[2] Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
[3] Blackett Laboratory, Imperial College, London SW7 2AZ, UK



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Example: 1d Quantum Ising Chain

Ising chain Hamiltonian $$\hat{H}_0(t) = -\sum_{n=1}^{N} \left[\sigma_n^x \sigma_{n+1}^x + g(t)\sigma_n^z\right]$$

Critical point $g_c = 1$

$g \ll 1 \qquad | \rightarrow\rightarrow\rightarrow \cdots \rightarrow\rangle$

$g \gg 1 \qquad | \uparrow\uparrow\uparrow \dots \uparrow\rangle$

$| \downarrow\downarrow\downarrow \dots \downarrow\rangle$

Excitations: $| \dots \uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow \dots \rangle$

Ways out of the Kibble-Zurek mechanism?

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Counterdiabatic driving: Ising Chain

Ising chain Hamiltonian $$\hat{H}_0(t) = -\sum_{n=1}^{N}\left[\sigma_n^x\sigma_{n+1}^x + g(t)\sigma_n^z\right]$$

Critical point $g_c = 1$

$g \ll 1$ $\quad |\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle$ $\qquad g \gg 1$ $\quad |\uparrow\uparrow\uparrow\dots\uparrow\rangle$

$|\downarrow\downarrow\downarrow\dots\downarrow\rangle$

Excitations: $|\dots\uparrow\downarrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\uparrow\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow\downarrow\dots\rangle$

Diagonalization: Jordan Wigner transformation + Fourier transform

$$\hat{H}_0(t) = 2\sum_{k>0}\Psi_k^\dagger\left[\sigma_k^z(g(t)-\cos k) + \sigma_k^x\sin k\right]\Psi_k$$

$$\hat{H}_1(t) = -\dot{g}(t)\sum_{k>0}\frac{1}{2}\frac{\sin k}{g^2+1-2g\cos k}\Psi_k^\dagger\sigma_k^y\Psi_k$$

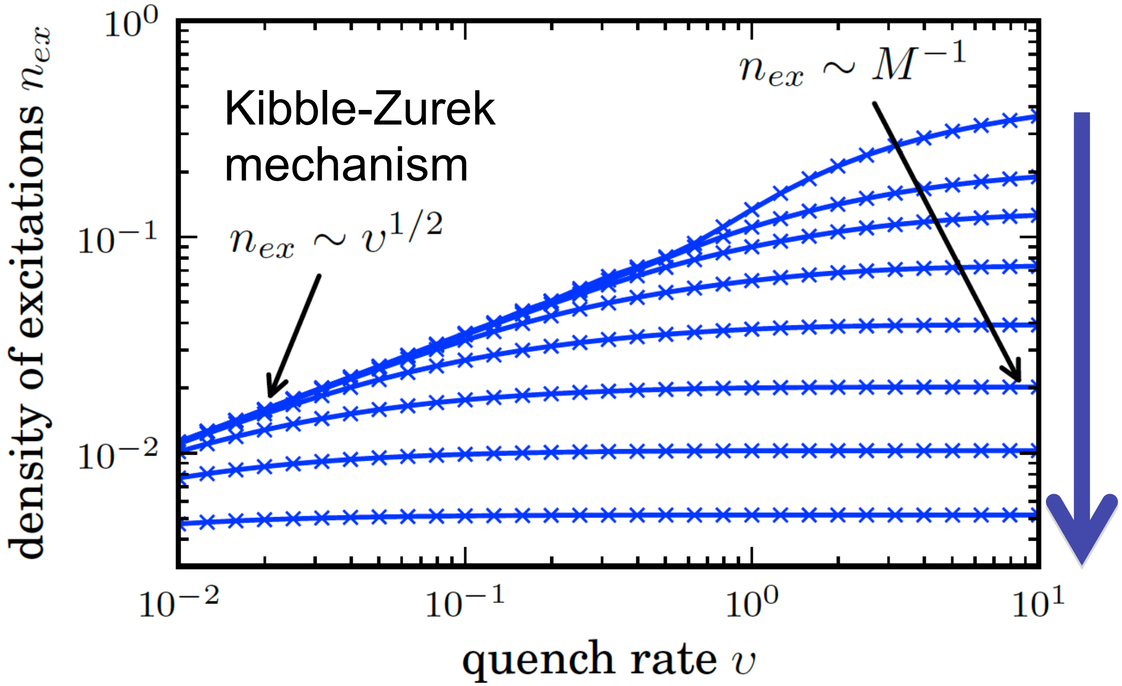
→ Long-range many-body interaction!

A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Truncated Auxiliary Hamiltonian

Quench through the critical point $g(t) = g_c - vt$

Truncated Auxiliary Hamiltonian



Range of auxiliary Hamiltonian

Suppressing Kibble-Zurek mechanism

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A. del Campo, M. M. Rams, W. H. Zurek, PRL 109, 115703 (2012)

Ultimate Quantum Speed Limits

Idea:

For unitary dynamics a time-energy uncertainty relation is known since 1945. What replaces it in open system dynamics?

Performance: Ultimate Quantum Limits

How fast can we go?

Not faster than the Quantum Speed Limit

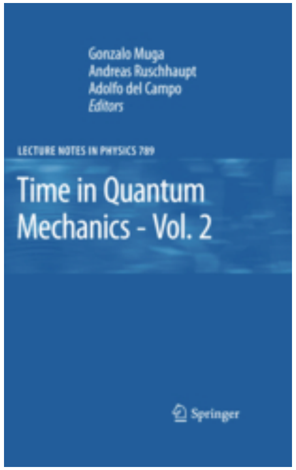
The speed at which a quantum state evolves is linked to the dynamics of the Hamiltonian

$$E = \langle \Psi | H | \Psi \rangle \qquad \Delta E = \sqrt{\langle \Psi | (H-E)^2 | \Psi \rangle}$$

Initial energy — Initial state — Energy variance

Minimum time required for a quantum state to evolve to an orthogonal state

$$T_{\mathrm{min}}(E, \Delta E) \equiv \max\left(\frac{\pi\hbar}{2E}, \frac{\pi\hbar}{2\Delta E}\right)$$



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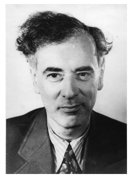
Time-energy uncertainty relation

Beautiful history

Passage time: Minimum time required for a state to reach an orthogonal state



Landau

Krylov

1945 Mandelstam and Tamm “MT”

$$\tau \geq \frac{\pi}{2}\frac{\hbar}{\Delta H}$$

1967 Fleming

1990 Anandan, Aharonov

1992 Vaidman, Ulhman

1993 Uffnik

1998 Margolus & Levitin “ML”

2000 Lloyd

2003 Giovannetti, Lloyd, Maccone: MT & ML unified

2003 Bender: no bounds in PT-symmetric QM

2009 Levitin,Toffoli

2012-2013 Bound for open (as well as unitary) system dynamics

Shortcuts at the speed limit?

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Time-energy uncertainty relation

2013
Taddei-Escher- Davidovich-de Matos Filho
AdC-Egusquiza-Plenio-Huelga
Deffner–Lutz

Rate of decay of the relative purity $$f(t) = \frac{\mathrm{tr}\,[\rho_0 \rho_t]}{\mathrm{tr}(\rho_0^2)}$$

Master equation $$\frac{d\rho_t}{dt} = \mathscr{L}\rho_t$$

Example: Markovian dynamics

$$\mathscr{L}\rho = -\frac{i}{\hbar}[H,\rho] + \sum_k \left(F_k \rho F_k^\dagger - \frac{1}{2}\left\{ F_k^\dagger F_k, \rho \right\} \right)$$

2012 MT-like bound for open (as well as unitary) system dynamics

$$f(t) = \cos\theta \qquad \tau_\theta \geq \frac{|\cos\theta - 1|\mathrm{tr}\rho_0^2}{\sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}} \geq \frac{4\theta^2 \mathrm{tr}\rho_0^2}{\pi^2 \sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}}$$

Bound to the velocity of evolution

$$v = \sqrt{\mathrm{tr}[(\mathscr{L}^\dagger \rho_0)^2]}$$

Time-energy uncertainty relation

Design of shortcuts at the quantum speed limit, i.e., saturating TEUR
In close & open quantum systems?

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Summary

Shortcuts to adiabaticity speed up processes by tailoring excitations

Noncritical systems

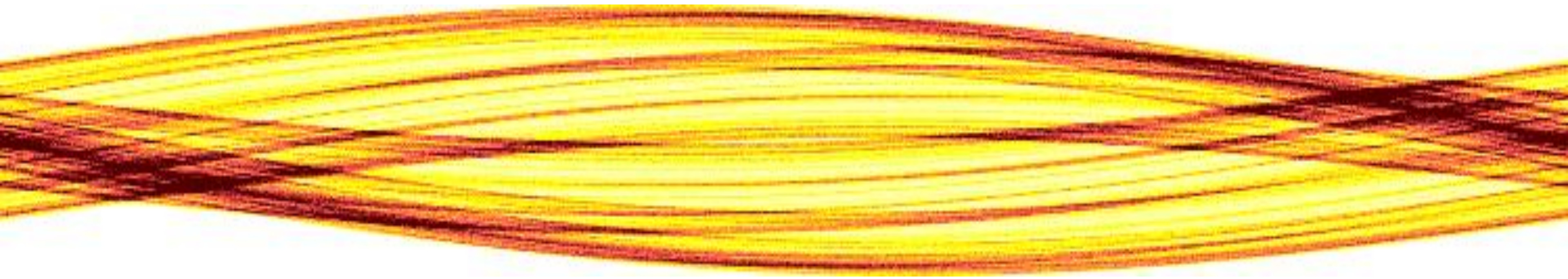
- Inverting Scaling laws
- Counterdiabatic driving
- Fast-forward technique

Noncritical systems

- The Kibble-Zurek mechanism
- Ways out: inhomogeneous driving, counterdiabatic driving

Ultimate quantum speed limits

Thanks for your attention!!



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